# Application of Total Variation Minimization to Doppler Tomography

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http://home.hiroshima-u.ac.jp/uemuram/dttvm/
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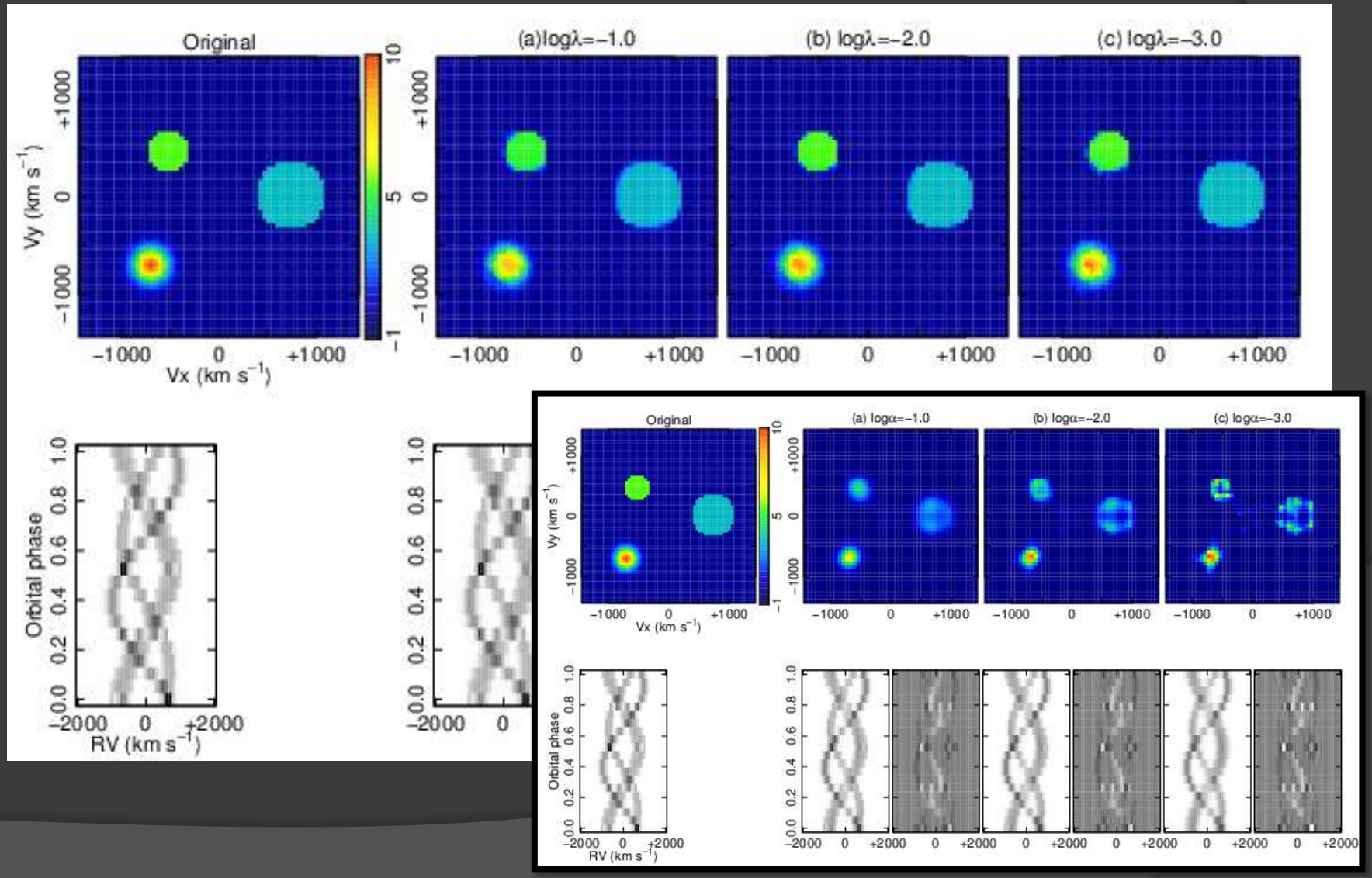
# Example

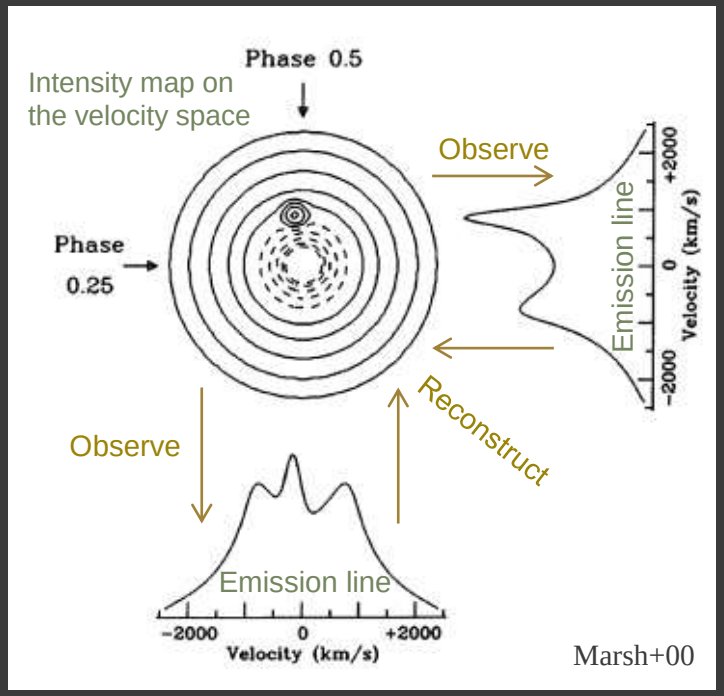

Original
(a) logλ=−1.0
(b) logλ=−2.0
(c) logλ=−3.0
Vy (km s$^{-1}$)
Vx (km s$^{-1}$)
+1000
0
−1000
10
5
−1
Orbital phase
RV (km s$^{-1}$)
−2000
+2000
0.0
0.2
0.4
0.6
0.8
1.0
Original
(a) logα=−1.0
(b) logα=−2.0
(c) logα=−3.0

# Outline

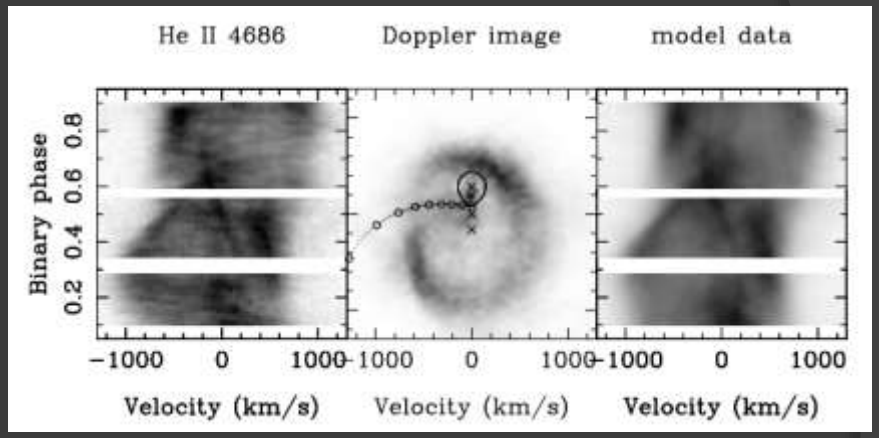
- Our model using total variation minimization.
- Tests with artificial and real data
- Cross-validation to determine a hyperparameter

# Doppler Tomography (DT)

(Marsh & Horne 1988)




IP Peg (Steeghs+97, Harlaftis+99)




- Data (Input)
  - Time variation in emission-line profiles
- Estimates (Output)
  - Intensity map in the velocity space

# DT as a linear problem

$$\hat{\boldsymbol{x}} = argmin \left\| \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} - \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\|_2^2 + \lambda f(\boldsymbol{x})$$

2nd order norm
= least-square term

Regularization term
Ex. MEM, TVM

- This inverse problem is usually an ill-posed problem
  - Filtering back projection
  - Introducing a regularization term (MEM? TVM?)

# MEM and TVM

- Maximum Entropy Method (MEM)
  - Standard method to date
  - Regularization:

$$S = -\sum_{i=1}^{M} p_i \ln \frac{p_i}{q_i}.$$

$$q_i = \frac{D_i}{\sum_{j=1}^{M} D_j},$$

  - MEM is statistically best, but physically best?
    - Hot spot and/or shock region may have sharp edges, making entropy low

- Total Variation Minimization (TVM)
  - Simple prior
  - Regularization:

$$TV(\boldsymbol{x}) = \sum \sqrt{(\Delta^h \boldsymbol{x})^2 + (\Delta^v \boldsymbol{x})^2}$$

  - Δx: differential operator = $x_{i+1} - x_i$
  - Sparse gradient

# Doppler Tomography using Total Variation Minimization (DTTVM)

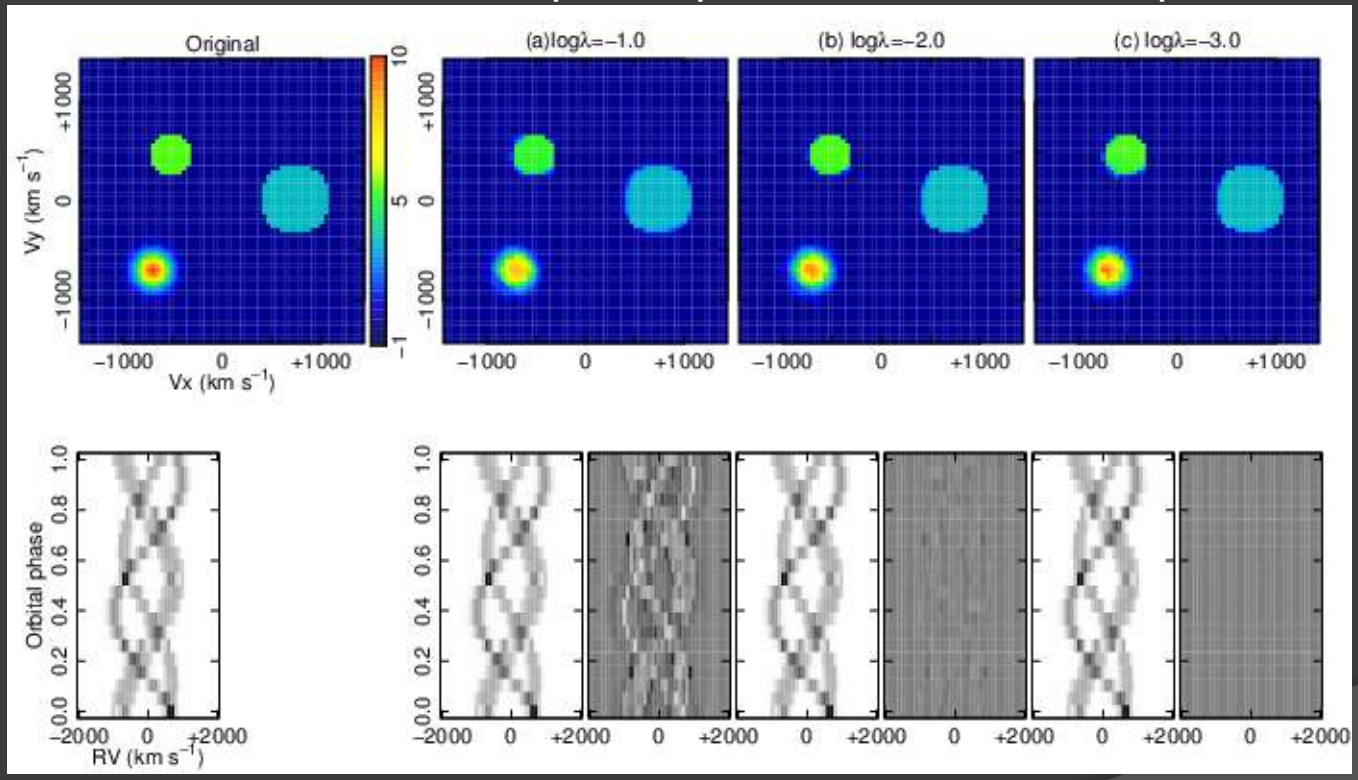
- Uemura+13, in prep.
- http://home.hiroshima-u.ac.jp/uemuram/dttvm/
- DT model: standard one
  - 2D, non-self absorbed
  - Instrument response: Gaussian
- Optimization by proximal gradient method
  - TwIST algorithm (Bioucas-Dias & Figueiredo 2007)
- Hyperparameter, λ can be given by hands, or determined by cross-validation
- Tests with artificial data
  - Input data: 630 points
  - Output map: 64x64 bin = 4096
  - Radial velocity resolution = 100 km/s
- Comparison with the MEM results
  - Calculation by the code in Spruit (1998)

$$\hat{\boldsymbol{x}} = argmin \left\| \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} - \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{pmatrix} \right\|_2^2 + \lambda f(\boldsymbol{x})$$

# Case 1: Three spots

large λ
= simple map

small λ
= complicated map

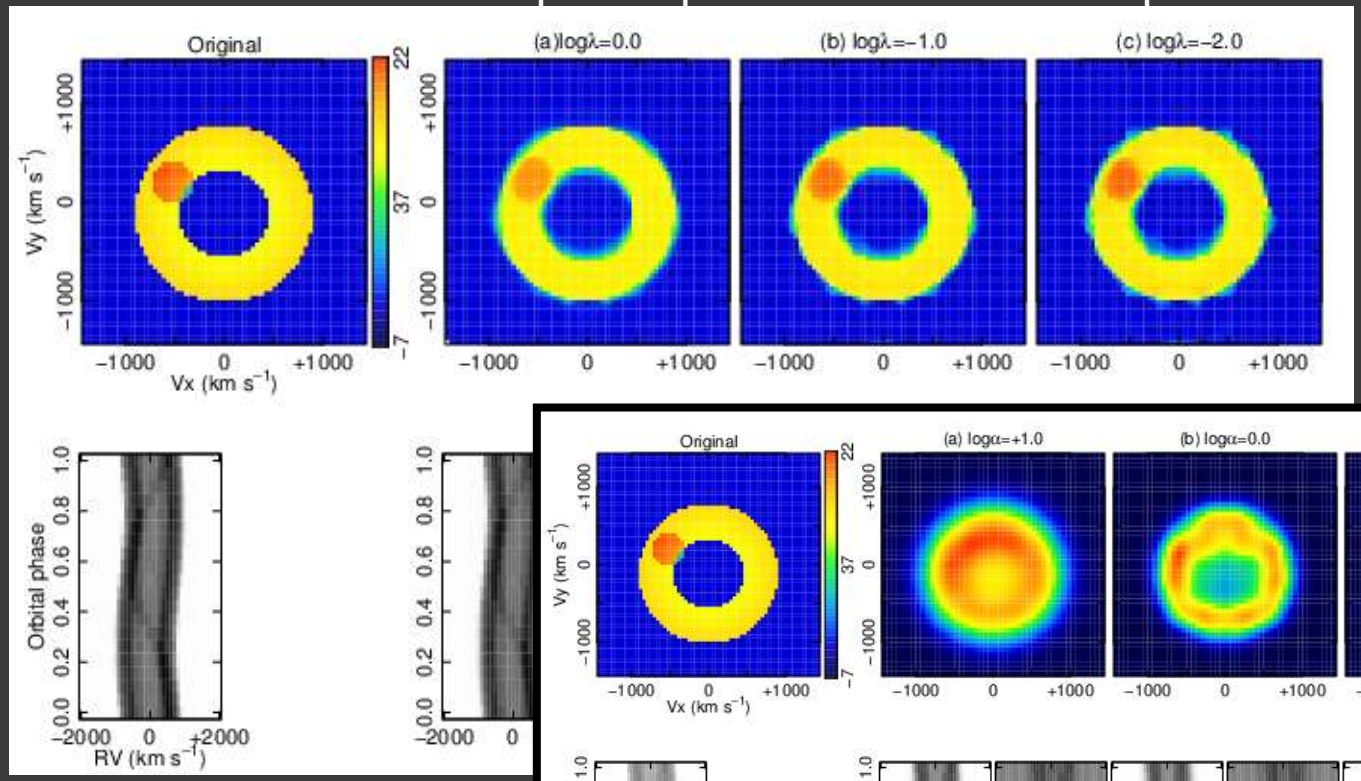



- ✓ The position, size, and structure of the spots are reproduced well.
- ✓ The residuals between the data and model spectra are smaller in smaller λ.

# Case 2: Disk + spot

large λ
= simple map

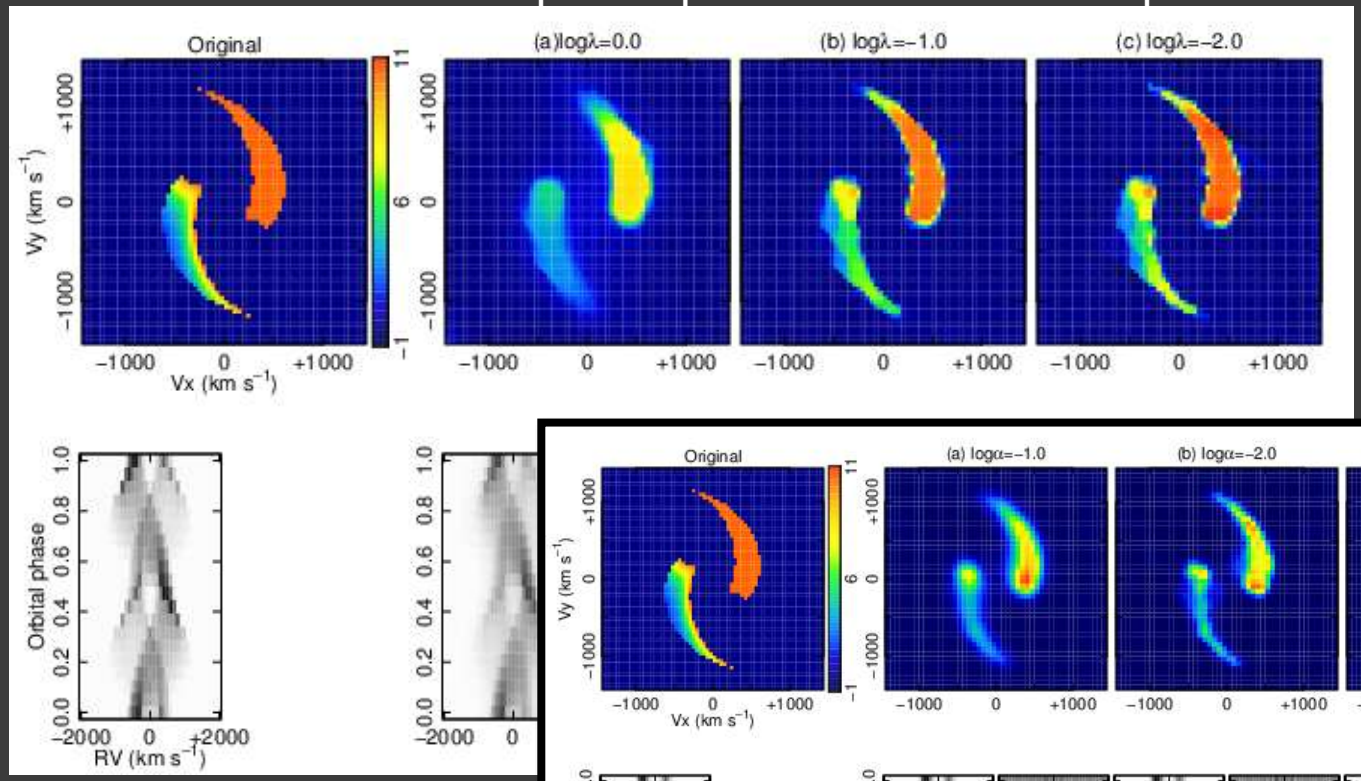
small λ
= complicated map



# Case 3: Spirals

large λ
= simple map

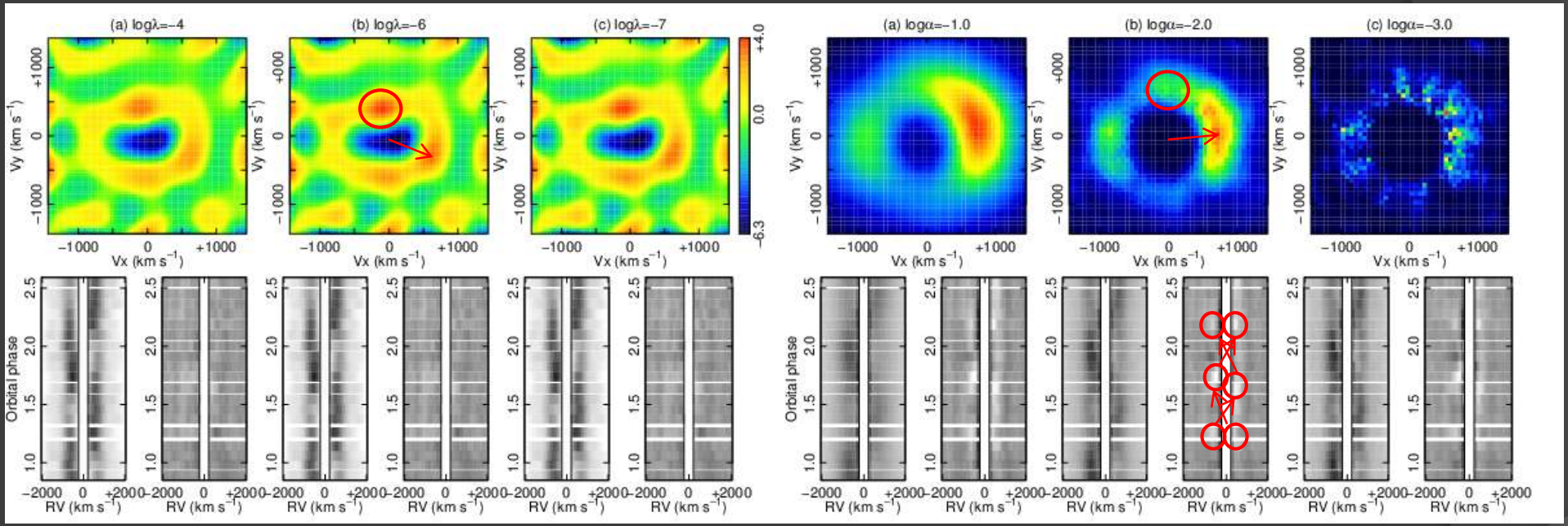
small λ
= complicated map




- ✓ 2 spirals having different structure
- ✓ Well reproduced
- ✓ The information of fine structures < ~20km is lost.

# Case 4: Data of WZ Sge

TVM MEM

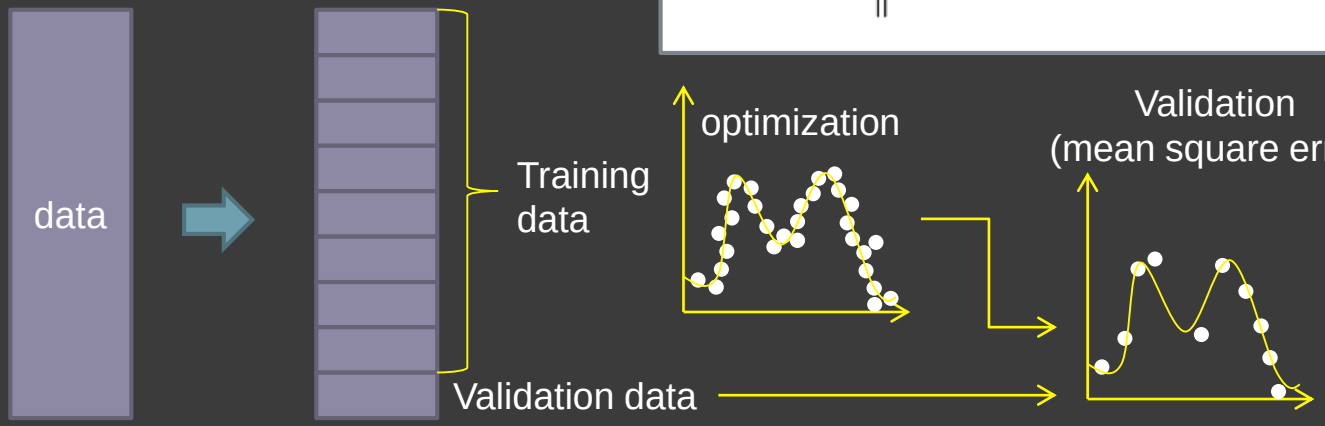



| | TVM | MEM |
|---|---|---|
| Disk structure | ellipse | circle |
| Strongest intensity | Phase ~ 0.1 | Phase = 0.0 |
| Secondary star? | strong | weak |

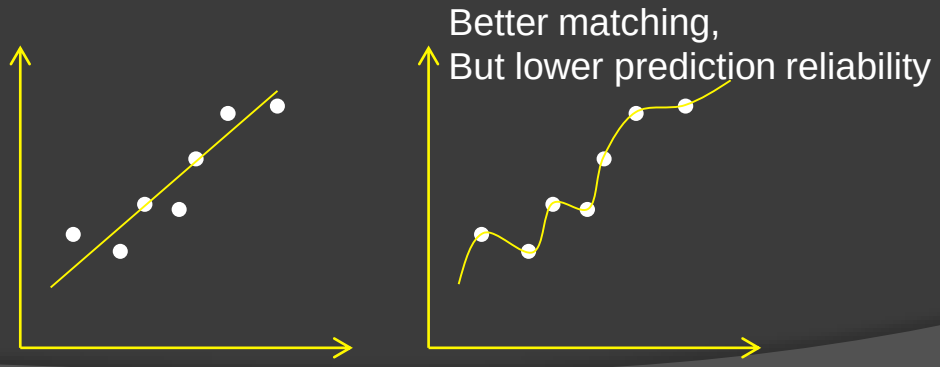
# How to determine the hyperparameter, λ

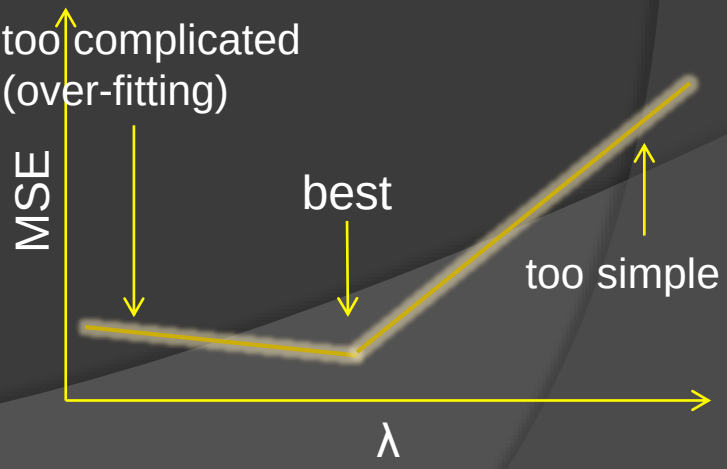
- Cross-validation

$$\hat{\boldsymbol{x}} = argmin \left\| \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} - \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\|_2^2 + \lambda f(\boldsymbol{x})$$




- Avoid over-fitting

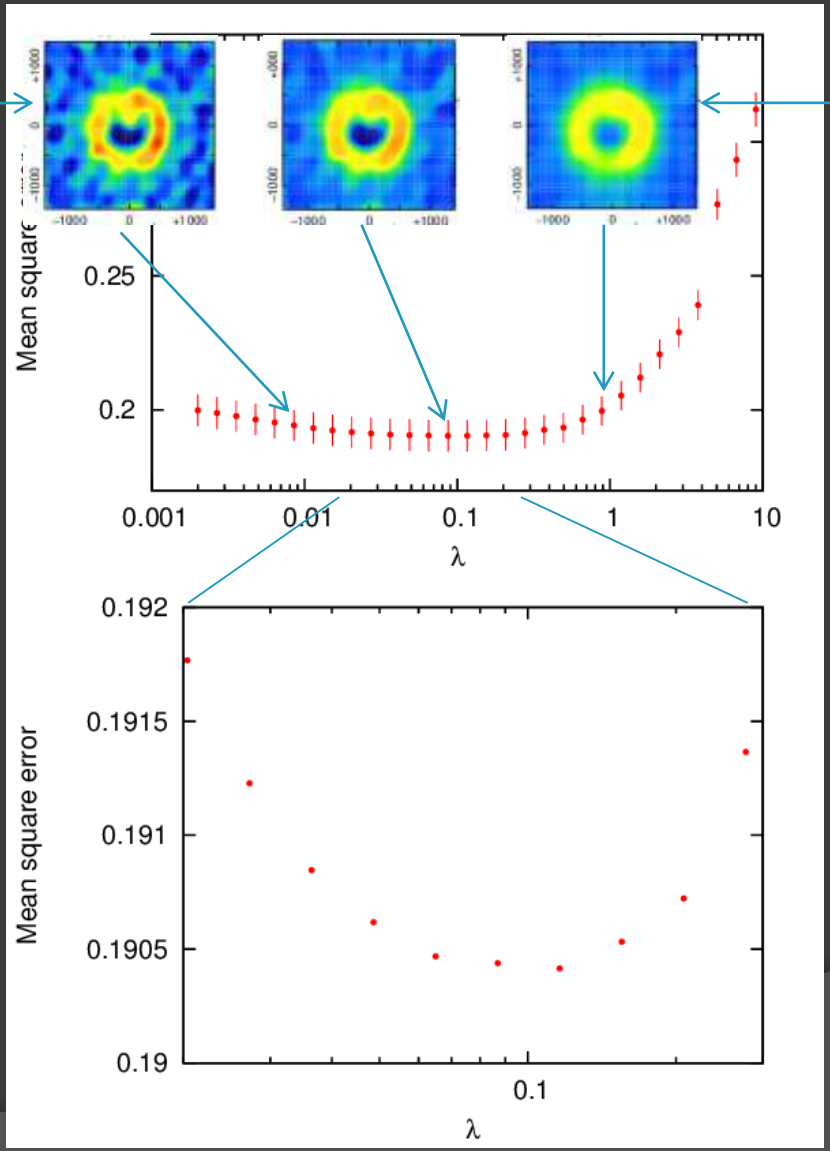


# Results of cross-validation (TU Men)

Small λ
Over-fitting

Large λ
Too simple



# Future plan

- Scientific motivation
  - No idea!
  - DTTVM may be a good tool if one can find rotating residuals in the MEM method.
- Application of TVM to another tomography
  - Eclipse mapping
  - And other…

# Summary

- We have developed the Doppler tomography using total variation minimization (DTTVM)
  - http://home.hiroshima-u.ac.jp/uemuram/dttvm/
- DTTVM provides a good tool to reconstruct localized and/or sharp edge features in Doppler maps.