Animation of Water Droplets on a Glass Plate

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ABSTRACT

This paper proposes a method for generating an animation of water droplets and their streams on a glass plate, such as a windowpane or windshield, taking into account the dynamics between fluid and solid. Water droplets run down an inclined glass plate if their masses are greater than a static critical weight. The streams from the droplets do not run straight down the glass plate but meander and some amount of water remains behind the flow due to the nature of the wetting phenomenon. Therefore, the mass of the water droplet decreases. When the mass becomes smaller than a dynamic critical weight, the flow stops. In this paper, a discrete model of a glass plate is developed to simulate the streams from the water droplets as described above. The glass plate is divided into small meshes. For rendering scenes through a glass plate upon which there are water droplets, we also develop a high-speed rendering method taking into account reflection and refraction of light. Instead of calculating the intersections between the ray and the objects, one of the most time-consuming processes in ray tracing, the proposed method determines pixel colors by using the intersection between the ray and a cuboid onto which objects in a scene are projected. Animations of rain droplets on a pane or windshield demonstrate the usefulness of the proposed method.

Keywords: Water Droplet, Flow, Discrete Model, Dynamics, Interfacial Tension

1. INTRODUCTION

A lot of methods have been developed for modeling and rendering fluid such as water in recent years. For rendering water surface such as the ocean, methods of modeling waves have been proposed (Max 1981; Mastin 1987; Peachey 1986; Fournier 1986; Ts'o 1987). To render more realistic images involving water, the color of water surfaces is calculated taking into account the radiative transfer of light in the water (Kaneda 1991a) and underwater glittering due to lens effect has also been rendered (Watt 1990). Methods of rendering fog (Nishita 1987; Kaneda 1991b) caused by vapor and wet road surfaces (Nakamae 1990) have also been proposed. In spite of these techniques, a lot of problems remain unaddressed because of the multiformity and the complex motion of water.

The animation of water droplets on a glass plate is vital for drive simulators and for displaying scenes through a windowpane on a rainy day. The shape and motion of droplets on a solid surface are under the sway of such factors as gravity, surface tension, interfacial tension, air resistance, and so on. Many efforts have been done to understand the phenomena between liquid and solid (de Gennes 1985; Janosi 1989). However, no computer graphics model considering these dynamics has previously been presented and water droplets and their streams down a pane have yet to be animated realistically.

Most of the water models developed for computer graphics (Max 1981; Mastin 1987) principally attempt to render large bodies of water without boundaries, such as the ocean. Therefore, they don't generate realistic scenes containing a seashore, where there is a boundary between liquid and solid. By taking into account the motion of water near boundaries, the realism of water animations has been improved (Peachy 1986; Fournier 1986; Ts'o 1987). However, these methods of modeling water cannot realistically animate the flow of very small amounts, because particles of water move in circular or ellipsoidal orbits around their initial positions in the models. To solve this problem, a method of modeling shallow water taking into account fluid dynamics has been presented (Kass 1990), when an animation of rain falling on a concave surface was screened. However, it is difficult to animate the streams of water drops using this model because of the absence of interfacial dynamics.

This paper proposes a method for generating a realistic animation of water droplets and their streams on a glass plate taking into account interfacial dynamics. As mentioned above, the shape and motion of water droplets on a glass plate depend on gravity to droplets, the respective surface tensions of the glass plate and the water, and the interfacial tension between them. Water droplets whose masses are greater than a static critical weight run down an inclined glass plate. They also tend to meander because of the roughness of the surface, any microscopical impurities on the surface, etc. The mass of the droplet running down decreases, because some amount of water remains behind the flow due to the nature of the wetting phenomenon. Finally, the mass becomes smaller than the dynamic critical weight and the flow stops.

To simulate the streams of water droplets described above, this paper proposes a discrete surface model of a glass plate. In the model, water droplets move from one grid to the next on the discrete surface. It is quite difficult to simulate the flow of water droplets for the purpose of high-precision engineering, because such a flow is a complicated process, where many parameters play a role (de Gennes 1985). Our main purpose is to generate a realistic animation, taking into account the dominant parameters of dynamical systems: gravity to water droplets, interfacial tensions, and the merging of water.

For reducing the calculation cost of animations containing scenes through a rainy windowpane or windshield, we also propose a high-speed rendering method taking into account reflection and refraction of light. The proposed method is an extension of environment mapping (Greene 1986) to be able to render water droplets efficiently. That is, objects in a scene are projected onto the planes of a cuboid, whose center is on the glass plate. Instead of calculating the intersections between the ray and objects, one of the most time-consuming processes in ray tracing, the proposed method generates an image by using pixel colors at the intersection between the ray and the cuboid.

In the following sections, methods for modeling the flow and shape of a water droplet and for high-speed rendering are described. Animations of rain droplets on a pane or windshield demonstrate the usefulness of the proposed method.

2. THE FLOW AND SHAPE OF WATER DROPLETS

The sticking and flowing of water droplets on a solid surface are very complicated phenomena and we need a lot of unmeasured parameters for their accurate simulation. As our main purpose is the generating of realistic animations, the proposed method is based on dominant parameters within the dynamics.

2.1 The Flow of Water Droplets

A water droplet begins to run down an inclined glass plate if the mass exceeds a static critical weight. This depends mainly on interfacial tensions and slightly on the slope of the glass (Janosi 1989). The route of the stream as it meanders down the plate is determined by impurities on the surface or in the droplet itself. Some amount of water remains behind because of the nature of the wetting phenomena. Therefore, the mass of the droplet decreases and finally the flow stops.

In order to animate water droplets and their stream as described above, the surface of the glass plate is divided into small meshes, and water droplets travel from one mesh point to the next. Figure 1 shows a $n \times n$ lattice on the glass plate, and an affinity for water, $f_{i,j}$ $(0 \le f_{i,j} \le 1)$, is assigned in advance to each lattice point. Consider a water droplet on a lattice point (i, j) whose mass is $m_{i,j}$. The droplet runs down if the mass satisfies the following equation:

$$m_{i,j} > m_c^s(\varphi) \; , \tag{1}$$

where $m_c^s(\varphi)$ is the static critical weight when the inclination angle of the surface is φ.

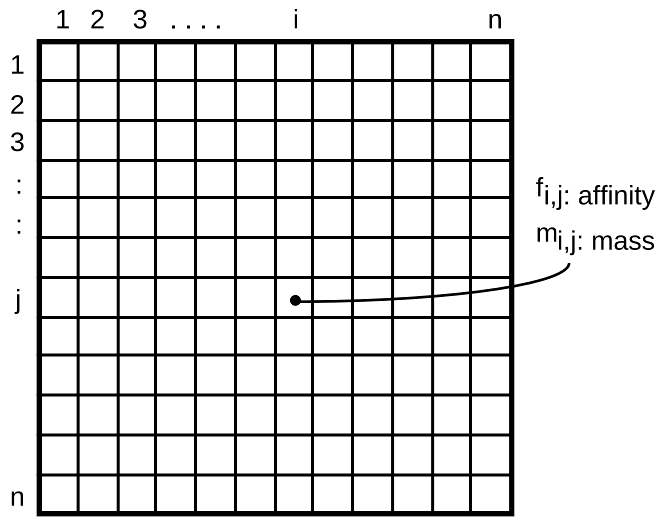

Fig. 1. Discrete surface model

A droplet at lattice point (i, j) moves to one of three points $(i-1, j+1)$, $(i, j+1)$, and $(i+1, j+1)$ to simulate the meandering. If no water exists at any of the three points, the droplet moves to the point where the following equation has the largest value.

$$d_{i+k,j+1} = w_1(\varphi, k)\, f_{i+k,j+1} , \qquad (k = -1, 0, 1) \tag{2}$$

where $d_{i+k,j+1}$ is the tendency for a water droplet move in direction $(i+k, j+1)$, $w_1(\varphi, k)$ is a parameter depending on an angle of the plate's inclination, φ, and its range is $0 < w_1(\varphi, k) \le 1$. If water exists in one of these three directions, then the droplet moves in the direction in which the water exists. In the case of it existing in more than two directions, the highest priority is direction $(i, j+1)$. If there is no water at point $(i, j+1)$, the droplet moves in the direction in which the water mass is larger.

The mass of the remaining water, $m'_{i,j}$, that is, the mass of a water droplet at point (i, j) at the next step depends on the affinity for water, $f_{i,j}$, at point (i, j):

$$m'_{i,j} = \begin{cases} m_{i,j} & (\text{if } m_{i,j} < f_{i,j}\, m_{\min}) \\ w_2\, f_{i,j} \left(m_{i,j} - f_{i,j}\, m_{\min}\right) + f_{i,j}\, m_{\min} & \left(\text{if } f_{i,j}\, m_{\min} \le m_{i,j} \le f_{i,j}\, m_{\min} + \dfrac{m_{\max} - m_{\min}}{w_2}\right) \\ f_{i,j}\, m_{\max} & \left(\text{if } m_{i,j} > f_{i,j}\, m_{\min} + \dfrac{m_{\max} - m_{\min}}{w_2}\right) \end{cases} , \tag{3}$$

where $m_{\min}$ and $m_{\max}$ are the maximum and minimum mass of water, respectively, that a lattice point hold, and w_2 is the parameter for remaining water. Then, at the next step, the mass of water, $m'_{i+k,j+1}$, at point $(i+k, j+1)$ $(k = -1, 0, 1)$ which the water ran into is given by the following equation:

$$m'_{i+k,j+1} = m_{i+k,j+1} + m_{i,j} - m'_{i,j} . \tag{4}$$

The speed of a running water droplet, v, doesn't depend on the mass of the droplet. It depends on the wetness of the direction $(i+k, j+1)$ and the angle of inclination of the glass plate φ.

$$v = v_0 + a_{i+k,j+1}(\varphi)\, t , \tag{5}$$

where v_0 is the speed when the droplet is put on the glass plate or just after the droplet collides with another, and $a_{i+k,j+1}(\varphi)$ is the acceleration of water droplets when the glass plate is inclined at φ degrees. If there is no water at the point $(i+k, j+1)$, $a_{i+k,j+1}(\varphi)$ is the acceleration, $a_d(\varphi)$, when a water droplet runs on a dry surface. If there is water, $a_{i+k,j+1}(\varphi)$ is the acceleration, $a_w(\varphi)$, when a water droplet runs on a wet surface ($a_w(\varphi) > a_d(\varphi)$ for the same angle φ). t is the time from when the droplet is put on the glass plate or from when it collides with another droplet. When two droplets whose mass and speed are m_1, m_2 and v_1, v_2, respectively, collide with each other and merge into one droplet with mass $m_1 + m_2$, the speed of the new droplet, v'_0, is calculated using the law of conservation of momentum:

$$v'_0 = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \ . \tag{6}$$

A running water droplet that has no water ahead decelerates and finally stops when the mass becomes smaller than the dynamic critical weight $m_c^d(\varphi)$. In this case, the speed of the droplet, v', is given by the following equation:

$$v' = v_0 - a'_d(\varphi)\, t \ , \tag{7}$$

where $a'_d(\varphi)$ is a deceleration of the droplet on the plate whose angle of inclination is φ, v_0 is the speed when the mass of the droplet becomes smaller than the dynamic critical weight, and t is the time from when the mass becomes smaller than the dynamic critical weight.

Using the algorithm described above, the positions and masses of all water droplets on an inclined glass plate are calculated for every frame of the animation.

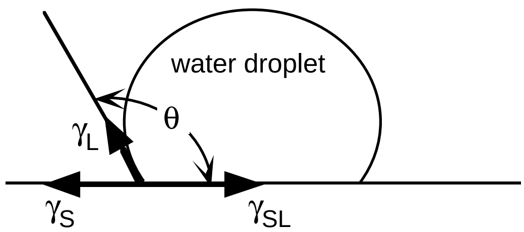

Fig. 2. Contact angle and interfacial tensions

2.2 The Shape of Water Droplets

Let's consider a water droplet on a horizontal glass plate. Interfacial tensions and the contact angle, θ, of water and the glass plate satisfy the following equation (de Gennes 1985) (see Fig. 2):

$$\gamma_{SL} - \gamma_S + \gamma_L \cos\theta = 0 \ , \tag{8}$$

where γ_{SL} is the interfacial tension between the glass and the water, and γ_S and γ_L are their respective surface tensions. Water spreads on the glass when $\gamma_S > \gamma_{SL} + \gamma_L$. In the case of this condition not being satisfied, a droplet on a plate in the stationary state has a shape, the contact angle of which θ satisfies Eq. 8.

In a dynamic state, that is, when a droplet is moving on the plate, the understanding of the shape of the droplet becomes more complicated than that in the stationary state. For example, a running droplet has different contact angles in its head and tail (advancing and receding contact angles, respectively) because of contact angle hysteresis (de Gennes 1985). This is because droplets have several contact angles which minimize its energy due to the roughness of the surface, uneven quality of the glass, and so on. Therefore, the contact angle changes from a locally stable angle to another as the droplet runs down. Furthermore, if the speed of the droplet is high, air resistance is no more a negligible factor with regard to the shape of the droplet. As mentioned above, the shape of a water droplet running down a plate depends on the roughness of the surface, the uneven quality of the solid, the speed of the droplet, and so on.

In this paper, our main purpose is not an accurate simulation but a realistic animation of water droplets and their streams. Moreover, the animation should be generated at a reasonable cost. For these reasons, we use a sphere to model a droplet on a plate. That is, a water droplet on a horizontal plate is modeled by the space which simultaneously satisfies the following two equations (see Fig. 3):

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \leq r^2 \tag{9}$$

$$z \geq r \cos\theta + z_0 \ , \tag{10}$$

where x_0, y_0, and z_0 are coordinates of the droplet's center, r is a radius of the droplet, and θ is the contact angle. The radius r of a water droplet with mass m is given by the following equation:

$$r = \left(\frac{3m}{\pi \, (2 + \cos^3\theta - 3\cos\theta)} \right)^{\frac{1}{3}} \tag{11}$$

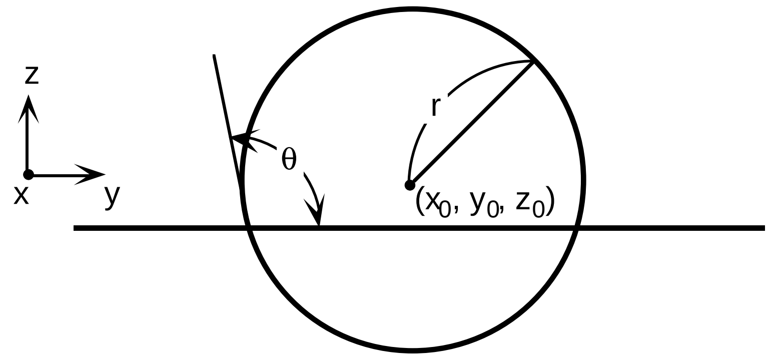

Fig. 3. Modeling a water droplet

2.3 Modeling of the Shape and Flow of Water Droplets

Using the method proposed in Section 2.1, the positions and masses of water droplets on an inclined glass plate are calculated for each frame of the animation. For each droplet with mass $m_{i,j}$, a radius $r_{i,j}$ is calculated by Eq. 11. Taking into account contact angle hysteresis, the contact angle of running droplets (i, j) with no water in front of them, i.e., (i–1, j+1), (i, j+1), or (i+1, j+1), is set to the advancing contact angle θ_A, and the contact angle of droplets following the top droplet is set to the receding contact angle θ_R ($0 < \theta_R < \theta_A$). If droplets overlap, their shape is modeled as a union of these regions.

3. RENDERING THE WATER DROPLETS

Ray tracing (Whitted 1980) is commonly used to render transparencies taking into account reflection and refraction of light. In spite of its many advantages, this method requires a lot of calculation time. To solve this problem, many improved methods have been proposed (Arvo 1989). However, in making an animation, a lot of calculation time is necessary, because thirty images per a second are required.

In this paper, we propose an extended method of environment mapping (Greene 1986) for rendering scenes through a glass plate with water droplets with a view to generating a realistic animation as cheaply as possible. Instead of calculating the intersections between rays and objects, the proposed method generates an image using background textures calculated in advance.

The proposed method can be outlined as follows:
(1) Background textures are calculated by projecting objects in the scene onto the faces of a cuboid whose center is on a glass plate.
(2) Calculating the direction of rays reflected or refracted by water droplets on a glass plate.
(3) Determining pixel colors by using background textures and the intersection of the ray and the cuboid.

3.1 Generating Background Textures

The cuboid used for generating background textures is set as follows. As shown in Fig. 4, the center of the cuboid is located at the center of the glass plate, and the front face, F_f, of the cuboid coincides with the projection plane. To generate the six background textures, the viewpoint is set at the center of the cuboid, and objects are projected onto its six faces. The resolution of the background texture on the front face is determined by that of the images in the animation. The resolution of the other background textures may be lower, because the number of rays reaching the other faces is fewer than the number reaching the front one.

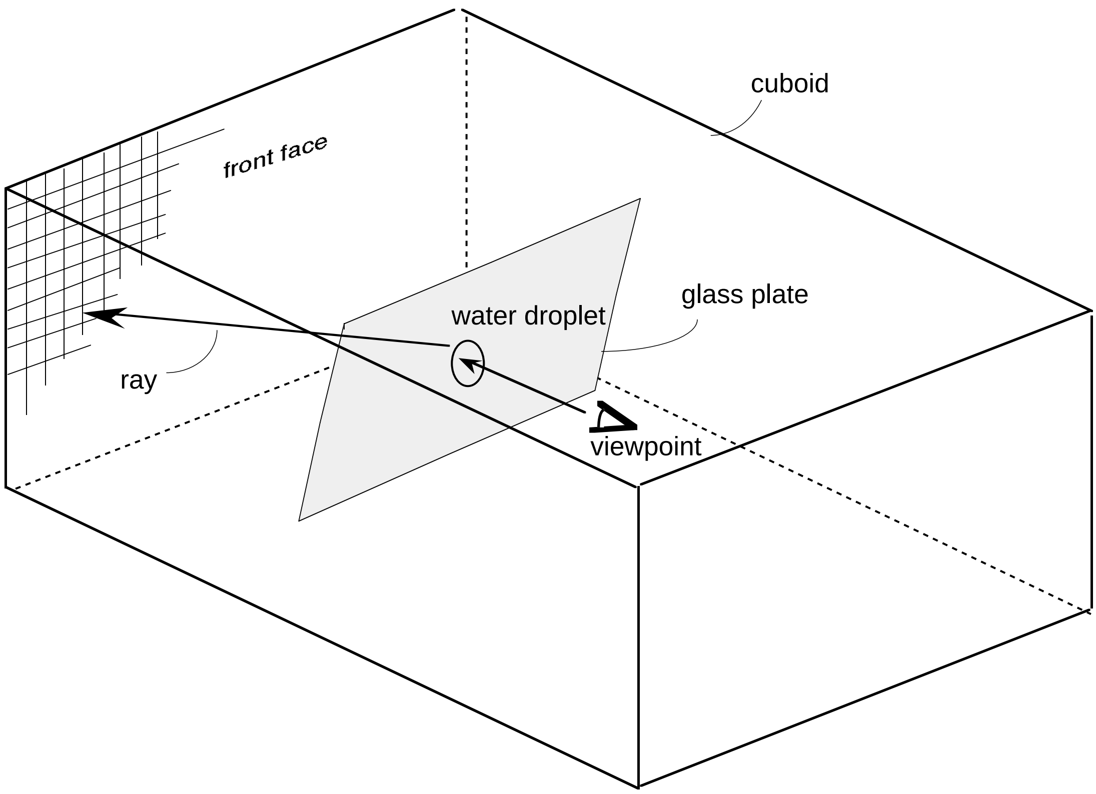

Fig. 4. A cuboid for generating background textures

3.2 Calculation of the Direction of a Ray

A ray of light incident on a transparent object such as water and glass splits into two directions; transmission and reflection. However, in the case of water and glass, the range of the incident angle that gives almost the same amount of light in these two directions of transmission and reflection is very narrow. For example, Fig. 5 shows a transmission coefficient obtained from the Fresnel equation (Hardy 1932) when light travels from water into the air. Most of the light is either just transmitted or reflected over a wide range of incident angles. Therefore, rays are traced in only one direction, that being the one in which the larger amount of light travels.

Fig. 5. Transmission coefficient when light travels from water to the air

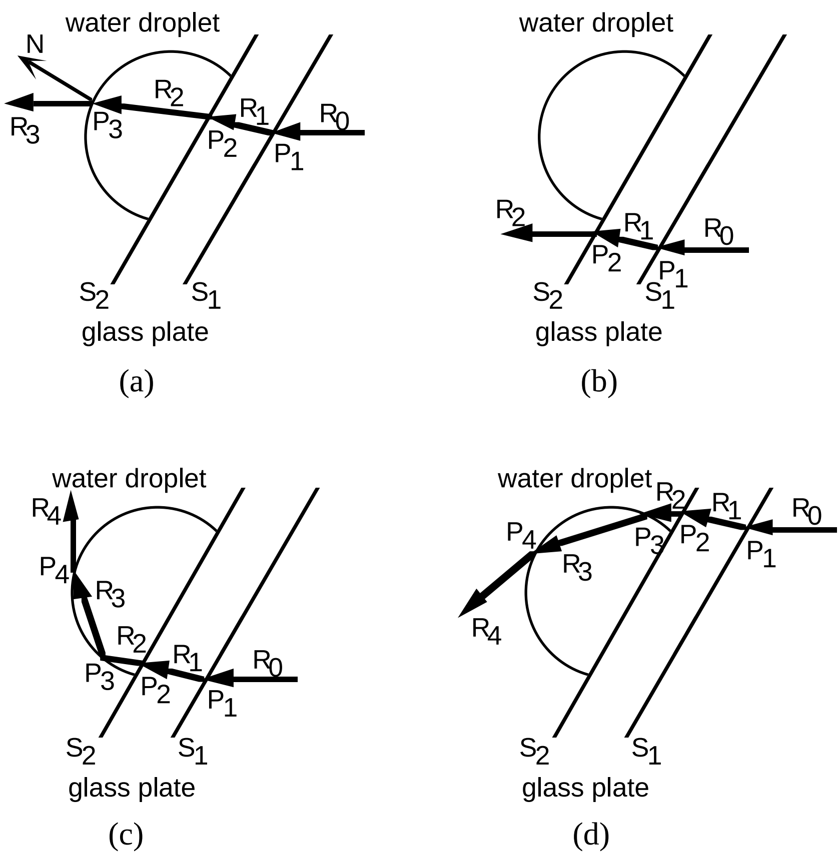

Fig. 6. Calculation of the direction of a ray

Let's consider a ray R_0 connecting the viewpoint and a point on the projection plane (see Fig. 6 (a)). First, the intersection, P_1, between the ray R_0 and the surface of the glass plate, S_1, is calculated. Using the normal of the surface S_1 and the index of refraction of glass, the direction of the refracted ray, R_1, is calculated. The intersection, P_2, between the ray R_1 and the other surface of the glass plate, S_2, is also calculated in the same manner. Next, using Eq. 9, it is checked whether the intersection P_2 is inside a water droplet or not. If it is, the intersection, P_3, between the ray R_2 and the surface of the droplet is calculated, and using the normal, N, of the surface and the index of refraction of water, the direction of the refracted ray, R_3, is calculated. If the intersection P_2 is outside a water droplet, the direction of the ray is calculated by using the normal of the surface S_2 (see Fig. 6 (b)).

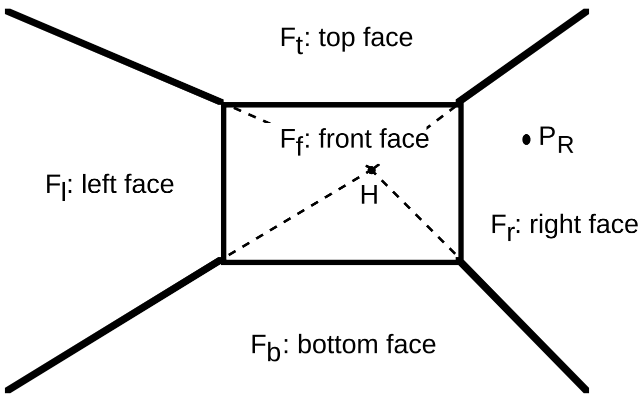

Fig. 7. Selecting a face of a cuboid

If the ray is reflected at the intersection P_3 (see Fig. 6 (c)) or the ray travels from the air into water (see Fig. 6 (d)), the direction of the ray away from a water droplet or the glass plate is calculated in the same manner described above.

3.3 Determining Pixel Colors

It is checked which faces of the cuboid a ray intersects with. If a ray goes forward, the face with which it intersects can be easily calculated by using the intersection, P_R, between the plane, F, including the front face of the cuboid, and the ray. That is, using the vertices of the front face F_f and a foot of a perpendicular, H, from the starting point of the ray calculated in the previous section to the plane F, the plane F is divided into five regions, as shown in Fig. 7. The face of the cuboid which intersects with the ray is determined by the region where the intersection P_R exists. For example, in the case of Fig. 7, the ray intersects with the right face F_r of the cuboid. If the ray goes backwards, the face can be determined in the same manner. The pixel color is determined by using a background texture corresponding to the face of the cuboid.

4. EXAMPLES

4.1 Flow of Water Droplets

Streams of water droplets on an inclined glass plate are simulated by using the proposed method. Figure 8 shows both the position and the mass of water droplets at four seconds after several droplets have been put on an inclined surface. Gradation of black and white is assigned to the mass of the droplets, and the larger the mass is, the brighter the color. The mass of the droplets put on the surface is larger from the right to the left side. The smaller the mass is, the shorter the stream, because the mass decreases to the dynamic critical weight due to in an earlier step.

Figure 9 shows the flow of water droplets every other second. Random amounts of water are fed every second from ten positions on the inclined surface. The figure shows that droplets follow the streams. In Fig. 10, random amounts of water are fed at random positions in advance. The droplets whose masses exceed the static critical weight start to run down, and a running droplet merges with another which is lying in its path. As time passes, the streams break, because droplets remaining behind gradually run down.

4.2 Animation of the Water Droplets

Figure 11 shows several frames of an animation rendered by using the proposed method described in Section 3. An outdoor scene is rendered through a windowpane on which water droplets are flowing. The angle of inclination of the windowpane is 90 degrees, and water is fed in the same way as in Fig.

9. Table 1 shows the parameters used in this animation. The boundary of the droplets is dark, because rays near the boundary are reflected by the surface of the droplet and do not go outside the room.

Figure 12 shows part of an animation on a rainy day. A street scene is rendered from the driver's seat of a car parked on a road. The inclination of the windshield is 35 degrees. Rain droplets are scattered in advance in the same way as in Fig. 10, and several droplets are fed every frame of the animation. Images containing a wet road surface (Nakamae 1990) are used as background textures. The resolution of the images is 512×395, and the rendering time per one frame is four minutes using a SiliconGraphics IRIS Indigo R4000.

Table 1. The parameters used in Fig. 11

φ	$m_c^s(\varphi)$	$m_c^d(\varphi)$	$w_1(\varphi, k)$		
90°	0.15	0.08	0.8 $(k = -1)$	1.0 $(k = 0)$	0.8 $(k = 1)$
$m_{\min}$	$m_{\max}$	w_2	$a_d(\varphi)$	$a'_d(\varphi)$	$a_w(\varphi)$
0.001	0.1	0.1	4.0	4.0	5.0
θ	θ_A	θ_R			
60°	70°	50°			

5. CONCLUSIONS

This paper proposes a method for generating an animation of water droplets and their streams on a glass plate taking into account interfacial dynamics. A discrete surface model is developed to simulate the stream of a water droplet running on a plane. This model makes it possible to animate water droplets meandering on an inclined glass plate.

Realistic images of water droplets can be generated taking into account the contact angles between water and solid. A simple method for rendering a scene through a wet windowpane is also proposed. Taking into account refraction and reflection of light, the proposed method generates an animation at a reasonable cost. Animations of rain droplets on a pane or windshield demonstrate the usefulness of the proposed method.

We need several parameters for modeling water droplets, and these parameters play an important role in creating a realistic animation. Some of the parameters are already measured, but some are not. To pursue a more realistic animation, research into these parameters should be undertaken.

A stream is modeled as a group of water droplets in the proposed method. It is suitable for an animation, because of its low calculation cost, but is not suitable for still, close-up images. A more sophisticated model of a stream, using mathematical surface patches such as Bezier and/or NURBS tensor products, is required for such a purpose.

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