EUROPEAN UNION

Structural Instruments 2014-2020

Competitiveness Operational Programme (COP)

nuclear physics

Extreme Light Infrastructure - Nuclear Physics (ELI-NP) - Phase II

Project Co-financed by the European Regional Development Fund

IFIN-HH

レーザープラズマ科学におけるBrown運動

Brownian Motion in High-Intensity Laser Science

as the sub-reader of

arXiv: 1611.05861 & arXiv: 1611.05458 (2016)

瀬戸慧大 / Keita SETO

ELI-NP / IFIN-HH

H28年度レーザープラズマ科学のための最先端シミュレーションコードの共同開発・共用に関する研究会


今日のお題は
電磁相互作用
(U(1)ゲージ)
高強度場
Brown運動
量子力学
高強度場物理の次世代模型として
Brown運動
による量子力学
K. Seto, “A Brownian Particle and Fields I”, arXiv: 1611.05861 (2016).
K. Seto, “A Brownian Particle and Fields II”, arXiv: 1611.05458 (2016).
瀬戸慧大, “古典物理から量子場へ: 放射の反作用”, プラ核学会誌(2017).

Seeking collaborators for …

By Keita Seto (ELI-NP/IFIN-HH)

Each topics can innovate the frontier of
high-intensity field physics (高強度場物理) !!

Physics side

- Pair creation/annihilation mechanism by Brownian model
- Photon-photon scatterings by Brownian model
- Applications of Brownian particles

Required skills: QED (QFT), stochastic analysis, etc.
(incl. measure theory, probability theory based on measure theory)

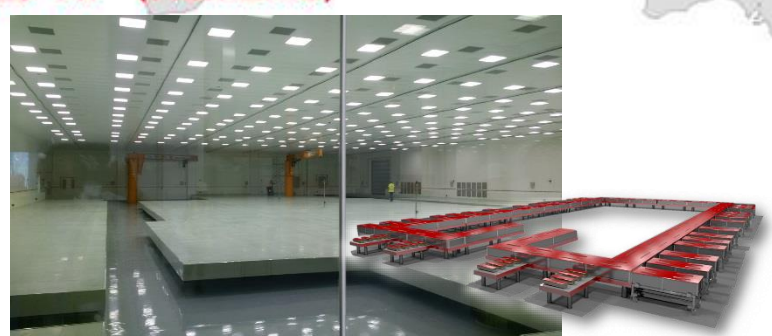
Simulation side

- Fokker-Planck equation
- Wiener process in Minkowski spacetime
  (Random value generation x 4 with the "Itô rule".)
- Integration of the above ideas + alpha

ELI-NP & Laser Facilities

http://www.icuil.org/

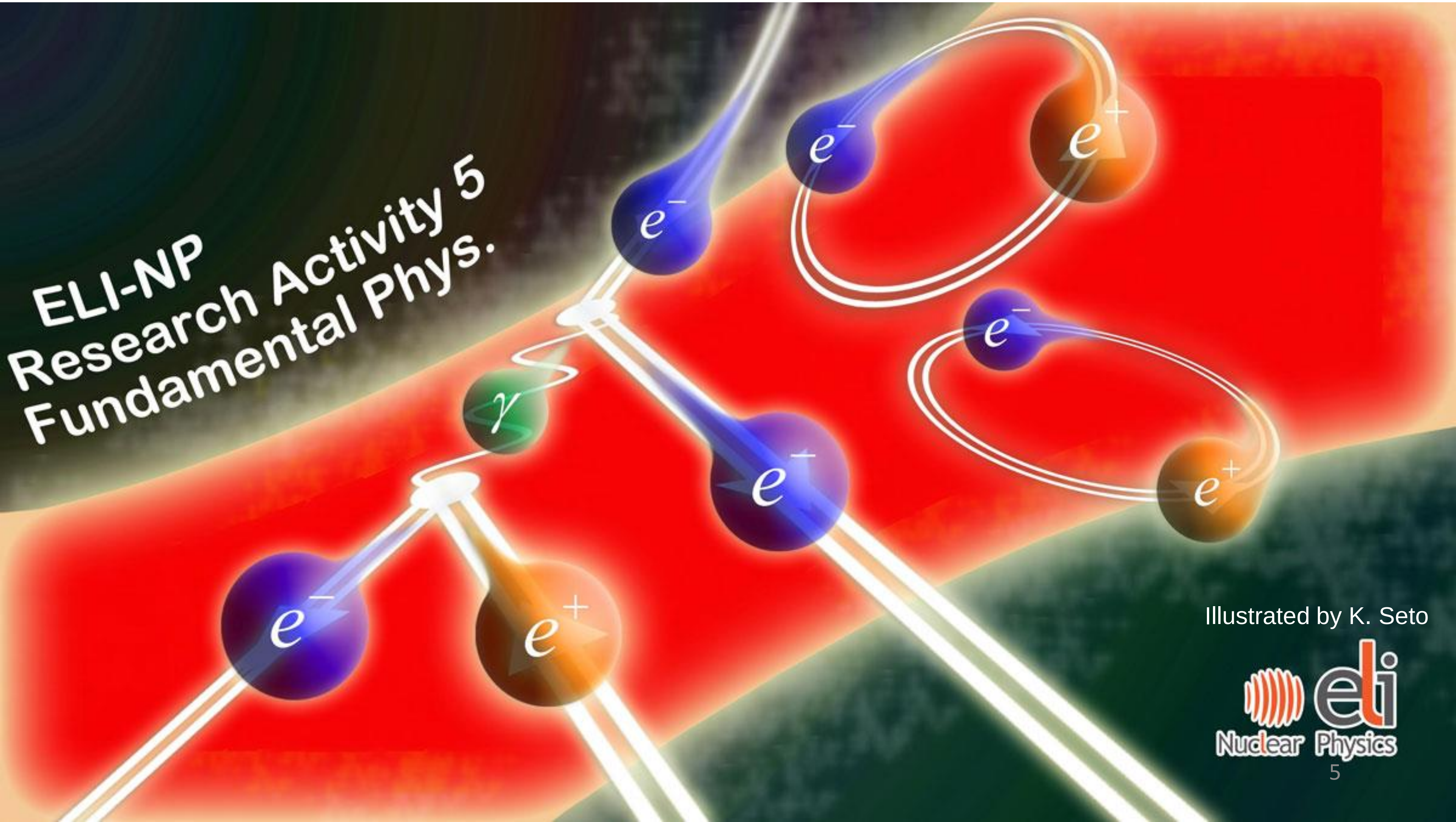
By Keita Seto (ELI-NP/IFIN-HH)



Non-linear QED @ ELI-NP

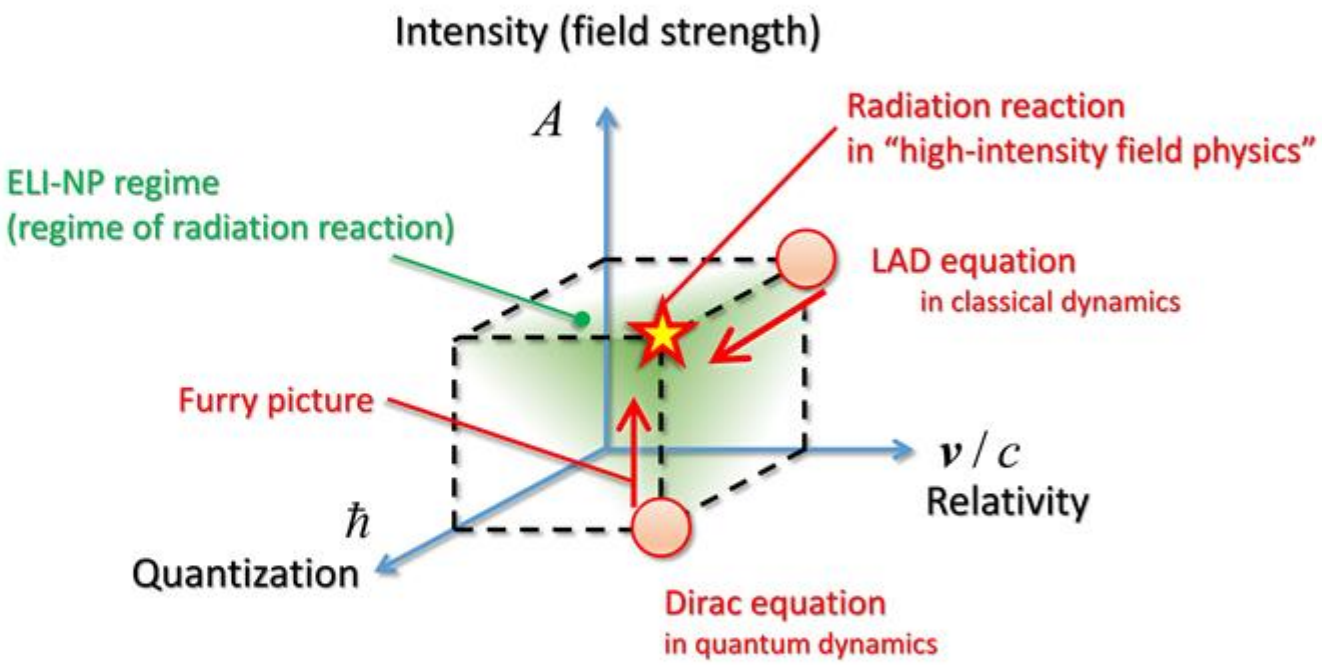
By Keita Seto (ELI-NP/IFIN-HH)

Radiation reaction = non-linear Compton scatterings ??



Present (Major) Schemes
Classical & Quantum Dynamics

By Keita Seto (ELI-NP/IFIN-HH)



LAD equation (classical dynamics)	$m_0 \frac{dv^\mu}{d\tau} = f_{\mathrm{ex}}{}^\mu + \frac{m_0 \tau_0}{c^2}\left(\frac{d^2 v^\mu}{d\tau^2} v^\nu - \frac{d^2 v^\nu}{d\tau^2} v^\mu \right) v_\nu$
Non-linear Compton scatterings	**Furry picture**

RA5-TDR:
Radiation Reaction @ E7 area

By Keita Seto (ELI-NP/IFIN-HH)

Investigation of
the running coupling
between an electron & radiation

$$e_{\mathrm{High-Field}} = q(\chi) \times e_{\mathrm{classical}}$$

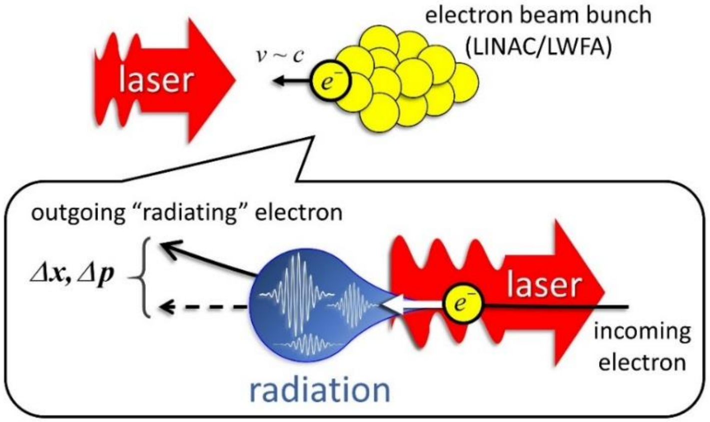
Solving Maxwell's eq

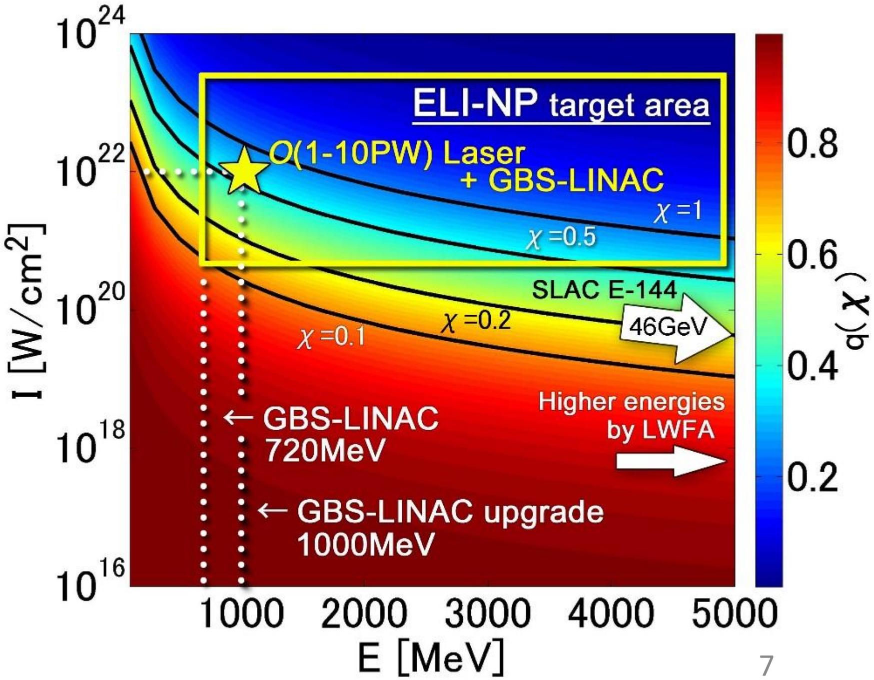
Radiation formula:

$$\frac{dW_{\text{High-intensity}}}{dt} = q(\chi) \times \frac{dW_{\text{classical}}}{dt}$$

$$\chi \propto E_{\text{electron}} \sqrt{I}$$

(Laser intensity dependence)





K. Seto, PTEP **2015**, 103A01 (2015).
K. Homma, et. al, Rom. Rep. Phys. **68 Supp.**, S233 (2016).
K. Seto, arXiv: 1611.05861 (2016)
K. Seto, arXiv: 1611.05458 (2016).

Non-linear Compton Scattering
in non-linear QED

By Keita Seto (ELI-NP/IFIN-HH)

Solve the Dirac equation with an external plane wave field strictly:

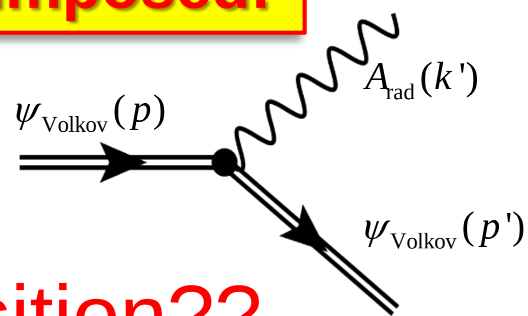
$$[\gamma_\mu (i\hbar\partial^\mu + eA^\mu_{\text{ex}}) - m_0 c \mathbb{I}^{4\times 4}]\psi_{\text{Volkov}}(x,p) = 0$$

Then, the Volkov solution is derived:

$$\begin{cases} \psi^{\pm}_{\text{Volkov}}(x,p,s) = e^{\mp\frac{i}{\hbar}\mathcal{S}^{\pm}[x,p;A,k]} \times \left[\mathbb{I}^{4\times 4} \mp \dfrac{e(\gamma_\mu k^\mu)\cdot(\gamma_\nu A^\mu_{\text{ex}})}{2p_\alpha k^\alpha}\right] \times \dfrac{u^{\pm}(p,s)}{\sqrt{2p^0}} \\ \mathcal{S}^{\pm}[x,p;A,k] = p_\mu x^\mu - \displaystyle\int^{\xi = k_\alpha x^\alpha} \frac{d\xi}{p_\alpha k^\alpha}\left(\pm e p_\nu A^\nu_{\text{ex}} + \frac{e^2 g_{\mu\nu} A^\mu_{\text{ex}} A^\nu_{\text{ex}}}{2}\right) \end{cases}$$

Orthogonality & Completeness are imposed:

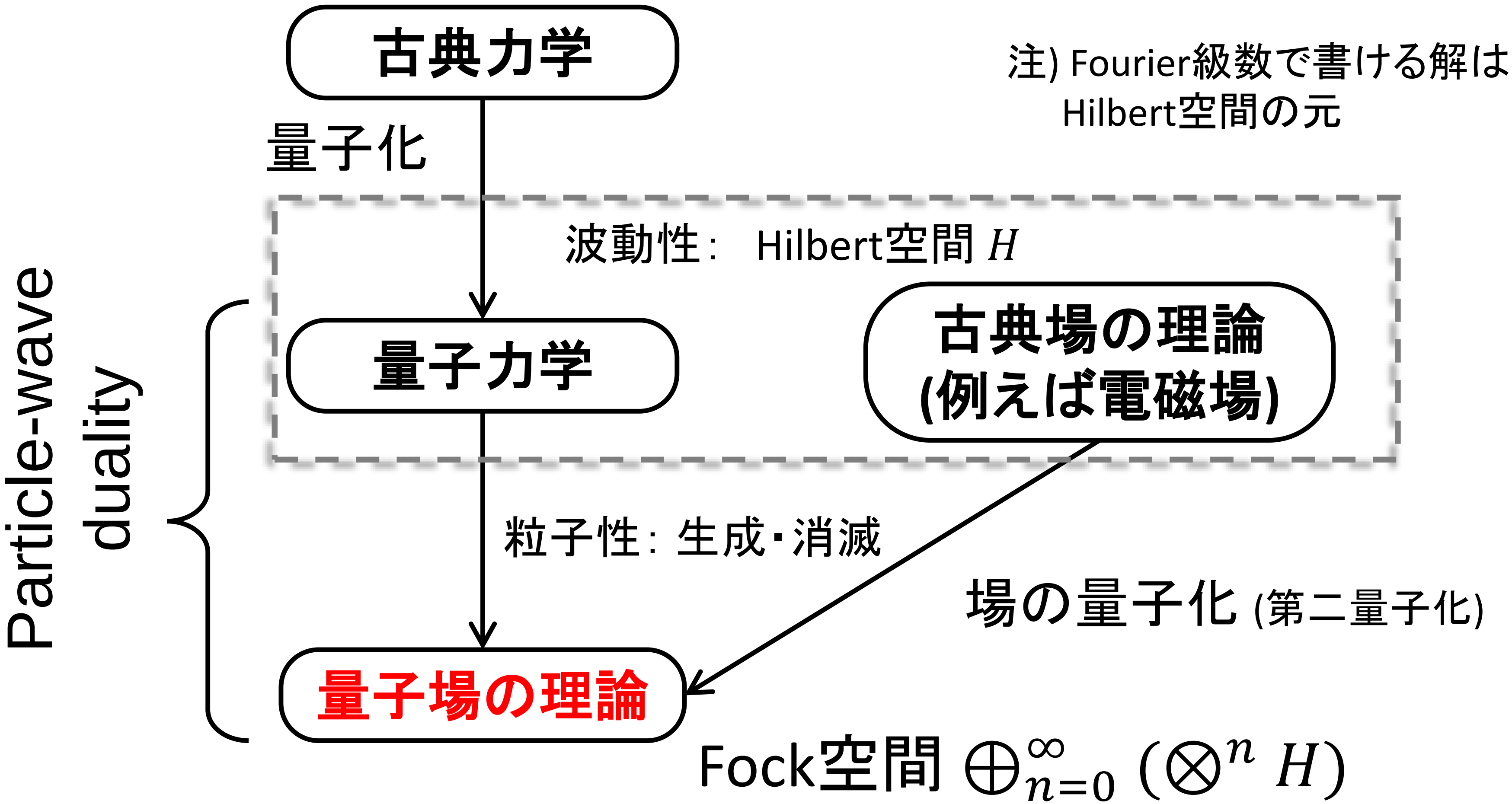
$$[\gamma_\mu (i\hbar\partial^\mu + eA^\mu_{\text{ex}} + e\boxed{A^\mu_{\text{rad}}}) - m_0 c \mathbb{I}^{4\times 4}]\psi(x,p) = 0 \quad \Rightarrow$$



But, focusing and superposition??

量子論は量子場まで駆け上がって初めて粒子と波の二重性を書ける

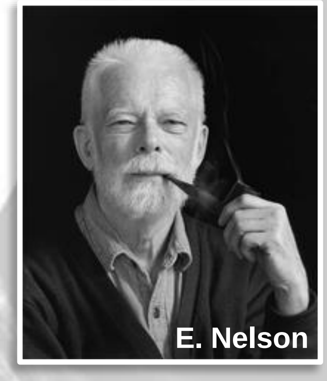
By Keita Seto (ELI-NP/IFIN-HH)



Non-linear Compton scattering は量子場の技巧が必須


量子論ってHilbert空間使ったり
粒子と波動の二重性を要求したり
考えるのがクソめんどくさい・・・

Idea: Ehrenfest's theorem



Schödinger equation

⇩

Wave function
$$\langle \boldsymbol{x}(t) \rangle := \int_{\mathbb{R}^3} d^3\boldsymbol{x}\ \psi^*(\boldsymbol{x})\hat{\boldsymbol{x}}\psi(\boldsymbol{x})$$

⇩

Average trajectory:
$$m_0 \frac{d^2\langle \boldsymbol{x}(t) \rangle}{dt^2} = \langle \boldsymbol{F}(x(t)) \rangle$$

⇧

Before taking the average…?

Nelson's "stochastic" mechanics !!

E. Nelsonによる Brown運動を使った量子力学 ①

By Keita Seto (ELI-NP/IFIN-HH)

数学者Edward Nelsonは次の方程式系がSchrödinger方程式と等価であることを示した。

Theorem (Nelson):

Probability space: $\left(\Omega, D(\mathscr{P}), \mathscr{P}\right)$

$\omega \in \Omega$

Kinematics: (Itô integral)

$$d\hat{\boldsymbol{x}}(\omega,t) = \boldsymbol{V}_{\pm}(\hat{\boldsymbol{x}}(t,\omega),t)dt + \sqrt{\frac{\hbar}{2m_0}}\, d\hat{W}_{\pm}(t,\omega)$$

Wiener process

Particle-wave duality

"particle"

Dynamics:

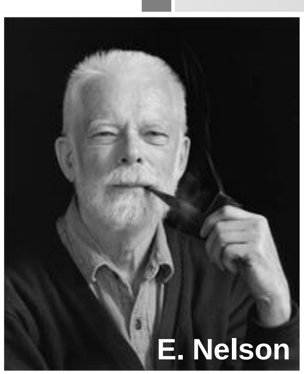
"Wave"

$$m_0\left[\begin{array}{c}\partial_t \boldsymbol{v}(\boldsymbol{x},t) + \boldsymbol{v}(\boldsymbol{x},t)\cdot\nabla\boldsymbol{v}(\boldsymbol{x},t) \\ -\boldsymbol{u}(\boldsymbol{x},t)\cdot\nabla\boldsymbol{u}(\boldsymbol{x},t) - \dfrac{\hbar}{2m_0}\nabla^2\boldsymbol{u}(\boldsymbol{x},t)\end{array}\right] = -\nabla V(\boldsymbol{x},t)$$

Sub-eqs:

$$\begin{cases} \boldsymbol{v}(\boldsymbol{x},t) = \dfrac{\boldsymbol{V}_+(\boldsymbol{x},t)+\boldsymbol{V}_-(\boldsymbol{x},t)}{2} = \mathrm{Im}\left\{\dfrac{\hbar}{m_0}\nabla\ln\psi(\boldsymbol{x},t)\right\} \\ \boldsymbol{u}(\boldsymbol{x},t) = \dfrac{\boldsymbol{V}_+(\boldsymbol{x},t)-\boldsymbol{V}_-(\boldsymbol{x},t)}{2} = \mathrm{Re}\left\{\dfrac{\hbar}{m_0}\nabla\ln\psi(\boldsymbol{x},t)\right\} \end{cases}$$

= Schrödinger方程式



Ehrenfest

古典論

$$\boldsymbol{v}(t) = \frac{d\mathbb{E}[\![\hat{\boldsymbol{x}}(t,\bullet)]\!]}{dt}dt$$

$$m_0\frac{d\boldsymbol{v}}{dt}(t) = -\nabla V(\mathbb{E}[\![\hat{\boldsymbol{x}}(t,\bullet)]\!],t)$$

E. Nelsonによる
Brown運動を使った量子力学 ②

Nuclear Physics

IFIN-HH

By Keita Seto (ELI-NP/IFIN-HH)

"Schrödingerの波動力学"に等価な"Nelsonの確率力学"のすごさ

① 完全な粒子の運動軌跡が描ける！(普通は書けて平均挙動まで)

$$d\hat{\boldsymbol{x}}(\omega,t)=\boldsymbol{V}_{\pm}(\hat{\boldsymbol{x}}(t,\omega),t)dt+\sqrt{\frac{\hbar}{2m_0}}d\hat{W}_{\pm}(t,\omega)$$

② 波動関数の二乗が確率密度であり、それはFokker-Planck方程式の解である

$$\partial_t p(\boldsymbol{x},t)+\nabla\cdot[\boldsymbol{V}_{\pm}(\boldsymbol{x},t)p(\boldsymbol{x},t)]\pm\frac{\hbar}{2m_0}\nabla^2 p(\boldsymbol{x},t)=0$$

⇩ ⇩

連続の式 浸透圧公式

$$\partial_t p(\boldsymbol{x},t)+\nabla\cdot[\boldsymbol{v}(\boldsymbol{x},t)p(\boldsymbol{x},t)]=0,\quad u(\boldsymbol{x},t)=\frac{\hbar}{2m_0}\times\nabla\ln p(\boldsymbol{x},t)$$

③ 量子・古典対応が式を見ただけで一発で分かる！

問題はこんな粒子が作る電流の定義とMaxwell方程式

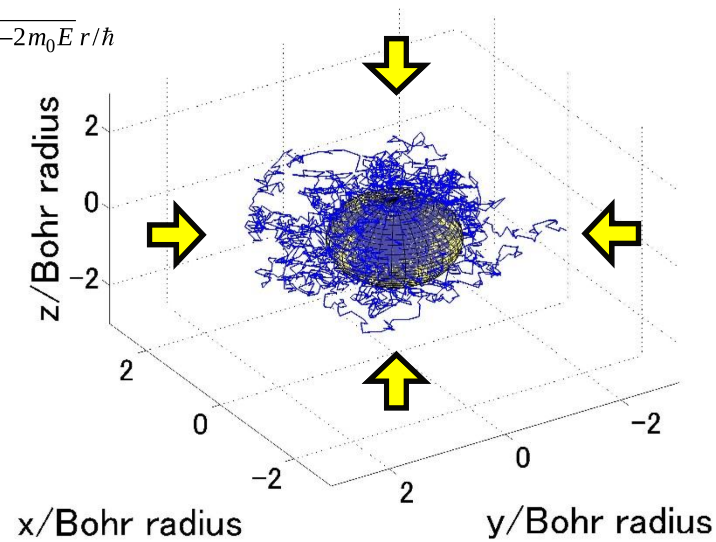
⇨ 放射の反作用(非線形・非摂動Compton散乱)には必須の知識

確率力学のすごさの例
- 水素原子周りの電子運動の計算 -

By Keita Seto (ELI-NP/IFIN-HH)

Schrödinger方程式:

$$\begin{cases} i\hbar\partial_t\psi(\boldsymbol{x},t)=\left[-\dfrac{\hbar^2}{2m_0}\nabla^2-\dfrac{e^2}{4\pi\varepsilon_0|\boldsymbol{x}-\boldsymbol{x}'|}\right]\psi(\boldsymbol{x},t) \\ \psi(\boldsymbol{x},t)=\mathrm{Const.}\times e^{-iEt/\hbar}\times r^l\times e^{-\sqrt{-2m_0E}\,r/\hbar} \\ E_l=-\dfrac{m_0e^4}{32\pi^2\varepsilon_0{}^2\hbar^2}\times\dfrac{1}{(l+1)^2} \end{cases}$$



⇨ $$\frac{\hbar}{m_0}\nabla\ln\psi(\boldsymbol{x},t)\Big|_{l=0}=\frac{\hbar}{m_0}\frac{\nabla\psi(\boldsymbol{x},t)}{\psi(\boldsymbol{x},t)}\Big|_{l=0}=-\sqrt{-\frac{2E_0}{m_0}}\nabla r$$

⇨ $$\begin{cases} \boldsymbol{v}(\boldsymbol{x},t)=\dfrac{\boldsymbol{V}_+(\boldsymbol{x},t)+\boldsymbol{V}_-(\boldsymbol{x},t)}{2}=\mathrm{Im}\left\{\dfrac{\hbar}{m_0}\nabla\ln\psi(\boldsymbol{x},t)\right\}=0 \\ \boldsymbol{u}(\boldsymbol{x},t)=\dfrac{\boldsymbol{V}_+(\boldsymbol{x},t)-\boldsymbol{V}_-(\boldsymbol{x},t)}{2}=\mathrm{Re}\left\{\dfrac{\hbar}{m_0}\nabla\ln\psi(\boldsymbol{x},t)\right\}=-\sqrt{-\dfrac{2E_0}{m_0}}\nabla r \end{cases}$$

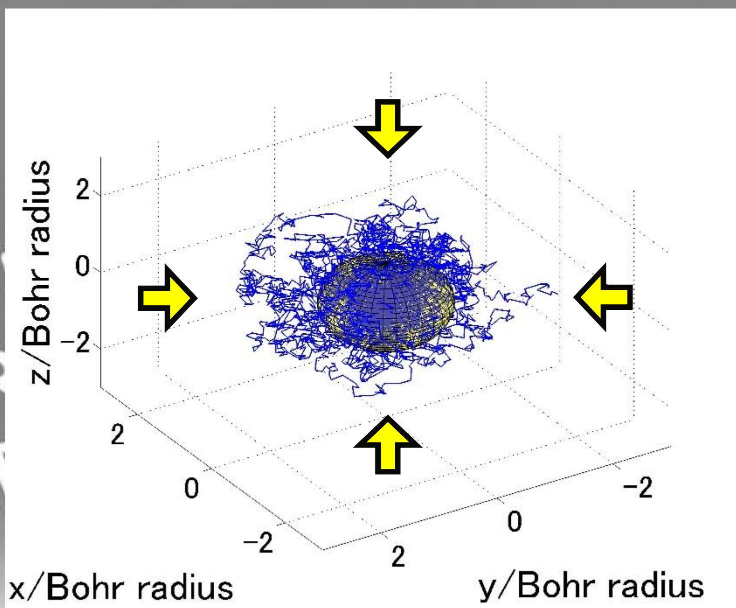
⇨ $$\begin{cases} \boldsymbol{V}_+(\boldsymbol{x},t)=-\dfrac{e^2}{4\pi\varepsilon_0\hbar}\times\nabla r \\ \nabla r=\dfrac{1}{r}\times(\boldsymbol{x}-\boldsymbol{x}') \end{cases}$$

⇨ $$\begin{aligned} d\hat{\boldsymbol{x}}(\omega,t)&=-\frac{e^2}{4\pi\varepsilon_0\hbar}\times\frac{\hat{\boldsymbol{x}}(\omega,t)-\boldsymbol{x}'}{|\hat{\boldsymbol{x}}(\omega,t)-\boldsymbol{x}'|}dt+\sqrt{\frac{\hbar}{2m_0}}d\hat{W}_\pm(t,\omega) \\ &=-\alpha\times\frac{\hat{\boldsymbol{x}}(\omega,t)-\boldsymbol{x}'}{|\hat{\boldsymbol{x}}(\omega,t)-\boldsymbol{x}'|}\times c\,dt+\sqrt{\frac{\hbar}{2m_0}}d\hat{W}_\pm(t,\omega) \end{aligned}$$

Relativistic Brownian motion under high-intensity laser field(s)

By Keita Seto (ELI-NP/IFIN-HH)

- Single scaler “Brownian” electron (Klein-Gordon equation)
- Fields (Maxwell equation)



+

From ドラえもん　25巻
竜宮上の八日間

Lorentz invariance !!
Field generation !!

A relativistic Brownian particle & Field generation mechanism

Nuclear Physics

IFIN-HH

By Keita Seto (ELI-NP/IFIN-HH)

Volume I は
相対論的Brown運動の
基本数学構成
について

preprint:ELI-NP/RA5-TDR 0003

A Brownian Particle and Fields I:
Construction of Kinematics and Dynamics

Keita Seto*

November 18, 2016

Extreme Light Infrastructure – Nuclear Physics (ELI-NP) /
Horia Hulubei National Institute for R&D in Physics and Nuclear Engineering (IFIN-HH),
30 Reactorului St., Bucharest-Magurele, jud. Ilfov, P.O.B. MG-6, RO-077125, Romania.

Tracking the "real" trajectory of a quantum particle still has been treated as the interpretation problem. It shall be expressed by the Brownian (stochastic) motion suggested by E. Nelson, but the well-defined ... radiation, which is equivalent to the Klein-Gordon particle and field system.

Keyword:
[Physics] Stochastic quantum dynamics, relativistic motion, field generation

This Volume I is reproduced from the parts of arXiv:1603.03379 .

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Volume II は
Brown運動に働く
放射の反作用
について

preprint:ELI-NP/RA5-TDR 0004

A Brownian Particle and Fields II:
Radiation Reaction as an Application

Keita Seto*

November 18, 2016

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Abstract

Radiation reaction has been investigated traditionally in classical dynamics and recently in non-linear QED for high-intensity field physics by high-intensity lasers. The non-linearity of QED is predicted ... **Volume II**, we aim the generalization of this applicable range by using the stochastic scalar electron model introduced in **Volume I**. We discuss the formulation of radiation reaction acting on a stochastic scalar electron and show the origin of this non-linearity correction in general conditions.

Keyword:
... motion, field generation
[mathematics] Applications of stochastic analysis

This **Volume II** is reproduced from a part of arXiv:1603.03379 .

1

The key issues are

- **the correspondence between classical and Brownian quantum dynamics**
- **the field generation mechanism**

by keeping the Lorentz invariance (relativistic covariance).

再 校

J. Plasma Fusion Res. Vol.93, No.1 (2017) ●-●

解説

古典物理から量子場の世界へ：放射の反作用

Radiation Reaction - from Classical Physics to Quantum Fields

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(原稿受付日：2016年9月8日)

Summary of Brownian particle model

Nuclear Physics eli
IFIN-HH
By Keita Seto (ELI-NP/IFIN-HH)

K. Seto, arXiv: 1611.05861 (2016).

Conclusion 18 (System of a scalar electron and a field). Consider the probability space $(\Omega, D(\mathscr{P}), \mathscr{P})$ and the Minkowski space $(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g), \mathscr{B}(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)), \mu)$. When the sub-$\sigma$-algebras of $\mathscr{P}_{\tau\in\mathbb{R}}$ and $\mathscr{F}_{\tau\in\mathbb{R}}$ with their filtration are included in $D(\mathscr{P})$, the D-progressive $\hat{x}(\circ,\bullet) \coloneqq \{\hat{x}(\tau,\omega) \in \mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g) | \tau \in \mathbb{R}, \omega \in \Omega\}$ characterized by

$$d\hat{x}^\mu(\tau,\omega) = \mathcal{V}^\mu_\pm(\hat{x}(\tau,\omega))d\tau + \lambda \times dW^\mu_\pm(\tau,\omega) \qquad (89)$$

kinematics

is defined as the kinematics of a stochastic scalar electron **[Definition 1]**. The following action integral **[Theorem 12]**

$$\begin{aligned}\mathfrak{S}[\hat{x}, \mathcal{V}, \mathcal{V}^*, A] = & \int_{\mathbb{R}} d\tau \int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)} d\mathfrak{M}(x,\tau)\, \frac{1}{2}\mathcal{V}^*_\alpha(x)\mathcal{V}^\alpha(x) \\ & - \int_{\mathbb{R}} d\tau \int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)} d\mathfrak{E}(x,\tau)\, A_\alpha(x)\mathrm{Re}\{\mathcal{V}^\alpha(x)\} \\ & + \int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)} d\mu(x)\, \frac{1}{4\mu_0 c}[F_{\alpha\beta}(x) + \delta f_{\alpha\beta}(x)] \cdot [F^{\alpha\beta}(x) + \delta f^{\alpha\beta}(x)] \qquad (90)\end{aligned}$$

dynamics

provides the following dynamics of a stochastic scalar electron **[Theorem 13]** and a field **[Theorem 15]** characterized by $\mathcal{V} \coloneqq (1-i)/2 \times \mathcal{V}_+ + (1+i)/2 \times \mathcal{V}_- \in \mathbb{V}^4_{\mathrm{M}} \oplus i\mathbb{V}^4_{\mathrm{M}}$ and $F \in \mathbb{V}^4_{\mathrm{M}} \otimes \mathbb{V}^4_{\mathrm{M}}$:

classical: $m_0 dv^\mu / d\tau = -e v_\nu F_{ex}^{\mu\nu}$

Scalar electron (Klein-Gordon eq.)

$$m_0 \mathfrak{D}_\tau \mathcal{V}^\mu(\hat{x}(\tau,\omega)) = -e\hat{\mathcal{V}}_\nu(\hat{x}(\tau,\omega))F^{\mu\nu}(\hat{x}(\tau,\omega)) \qquad (91)$$

Maxwell eq.

$$\partial_\mu [F^{\mu\nu}(x) + \delta f^{\mu\nu}(x)] = \mu_0 \times \mathbb{E}\left[\!\left[-ec\int_{\mathbb{R}} d\tau'\, \mathrm{Re}\{\mathcal{V}^\nu(x)\}\, \delta^4(x - \hat{x}(\tau',\omega))\right]\!\right] \qquad (92)$$

Here, the dynamics of (91) is equivalent to the Klein-Gordon equation. These dynamics fulfill the $U(1)$ gauge symmetry such that

$$\phi'(x) = e^{-ie\Lambda(x)/\hbar} \times \phi(x), \qquad A'^\alpha(x) = A^\alpha(x) - \partial^\alpha\Lambda(x) \quad . \qquad (93)$$

U(1) gauge symmetry

Sense by Mathematical physicist

Nuclear Physics
IFIN-HH
By Keita Seto (ELI-NP/IFIN-HH)

Let us consider the Mathematical spaces …

$\mathcal{B}(I)$ **The Borel σ-algebra of a topological space** I

1) Probability space

$$\left(\Omega, D(\mathscr{P}), \mathscr{P}\right)$$

⇩

D-progressive process (Relativistic kinematics)

$$d\hat{x}^\mu(\tau,\omega) = \mathcal{V}_\pm^\mu(\hat{x}(\tau,\omega))d\tau + \lambda \times dW_\pm^\mu(\tau,\omega)$$

⇩

Fokker-Planck eq.

$$\partial_\tau p(x,\tau) + \partial_\mu[\mathcal{V}_\pm^\mu(x)p(x,\tau)] \pm \frac{\lambda^2}{2}\partial^\mu\partial_\mu p(x,\tau) = 0$$

⇩

Proper time

$$d\tau = \frac{1}{c}\times\sqrt{\mathbb{E}[\![\hat{d}^*\hat{x}_\mu(\tau,\bullet)\cdot\hat{d}\hat{x}^\nu(\tau,\bullet)]\!]}$$

Kinematics

$$dx^\mu = v^\mu(\tau)d\tau$$

2) Metric Affine space (Minkowski spacetime)

$$\left(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g), \mathcal{B}(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)), \mu\right)$$

⇩

Action integral

$$\mathfrak{S}[\hat{x},\mathcal{V},\mathcal{V}^*,A] = \int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)} d\mu(x)\,\mathfrak{L}(x,\hat{x},\mathcal{V},\mathcal{V}^*,A)$$

⇩

EOMs

$$m_0\mathfrak{D}_\tau\mathcal{V}^\mu(\hat{x}(\tau,\omega)) = -e\hat{\mathcal{V}}_\nu(\hat{x}(\tau,\omega))F^{\mu\nu}(\hat{x}(\tau,\omega))$$

$$\partial_\mu[F^{\mu\nu}(x)+\delta f^{\mu\nu}(x)] = \mu_0\times\mathbb{E}\left[\!\!\left[-ec\int_{\mathbb{R}} d\tau\,\mathrm{Re}\left\{\mathcal{V}^\nu(x)\right\}\delta^4(x-\hat{x}(\tau,\bullet))\right]\!\!\right]$$

Dynamics

$$\frac{dv^\mu}{d\tau} = -\frac{e}{m_0}v_\nu F^{\mu\nu}$$

Kinematics
D-progressively measurable process

By Keita Seto (ELI-NP/IFIN-HH)

K. Seto, arXiv: 1611.05861 (2016).

Definition 1 (D-progressive $\hat{x}(\circ,\bullet)$). Consider the $\{\mathscr{P}_\tau\}$-progressively measurable and the $\{\mathscr{F}_\tau\}$-progressively measurable process $\hat{x}(\circ,\bullet)$.

[Nelson's (S1)] For each $(\tau,\omega)\in\mathbb{R}\times\Omega$, when the following $\mathscr{B}((-\infty,\tau])\times\mathscr{P}_\tau$ measurable function $\mathcal{V}_+^\mu(\hat{x}(\circ,\bullet))$ and the $\mathscr{B}([\tau,\infty))\times\mathscr{F}_\tau$ measurable function $\mathcal{V}_-^\mu(\hat{x}(\circ,\bullet))$ exist as the limit in L^1, $\hat{x}(\circ,\bullet)$ is named "Nelson's (S1)-process" [2]:

$$\mathcal{V}_+^\mu(\hat{x}(\tau,\omega)) = \lim_{\delta t\to 0+}\mathbb{E}\left[\!\!\left[\frac{\hat{x}^\mu(\tau+\delta\tau,\bullet)-\hat{x}^\mu(\tau,\bullet)}{\delta\tau}\right|\mathscr{P}_\tau\right]\!\!\right](\omega) \tag{1}$$

$$\mathcal{V}_-^\mu(\hat{x}(\tau,\omega)) = \lim_{\delta t\to 0+}\mathbb{E}\left[\!\!\left[\frac{\hat{x}^\mu(\tau,\bullet)-\hat{x}^\mu(\tau-\delta\tau,\bullet)}{\delta\tau}\right|\mathscr{F}_\tau\right]\!\!\right](\omega) \tag{2}$$

[D-progressive] Let $W_+(\circ,\bullet)$ and $W_-(\circ,\bullet)$ be the forward and backward standard Wiener processes. For a given set $(\tau,\omega)\in\mathbb{R}\times\Omega$ with respect to $\tau_a\le\tau\le\tau_b$, consider the following $\{\mathscr{P}_\tau\}$-progressive and $\{\mathscr{F}_\tau\}$-progressive Itô process [25].

$$\begin{aligned}\hat{x}^\mu(\tau,\omega) &= \hat{x}^\mu(\tau_a,\omega)+\int_{\tau_a}^{\tau}d\tau'\,\mathcal{V}_+^\mu(\hat{x}(\tau',\omega))+\lambda\times\int_{\tau_a}^{\tau}dW_+^\mu(\tau',\omega) &(3)\\ &= \hat{x}^\mu(\tau_b,\omega)-\int_{\tau}^{\tau_b}d\tau'\,\mathcal{V}_-^\mu(\hat{x}(\tau',\omega))-\lambda\times\int_{\tau}^{\tau_b}dW_-^\mu(\tau',\omega) &(4)\end{aligned}$$

Where, $\lambda := \sqrt{\hbar/m_0}\in\mathbb{R}$ [19]. This stochastic process includes Nelson's (S1)-process obviously. Then, introduce the modified rule of Nelson's (S2) and (S3)-processes [2] as the limit in L^2:

$$g = -\lim_{\delta t\to 0+}\mathbb{E}\left[\!\!\left[\frac{[W_+(\tau+\delta\tau,\bullet)-W_+(\tau,\bullet)]\otimes[W_+(\tau+\delta\tau,\bullet)-W_+^\nu(\tau,\bullet)]}{\delta\tau}\right|\mathscr{P}_\tau\right]\!\!\right](\omega) \tag{5}$$

$$g = +\lim_{\delta t\to 0+}\mathbb{E}\left[\!\!\left[\frac{[W_-(\tau,\bullet)-W_-(\tau-\delta\tau,\bullet)]\otimes[W_-(\tau,\bullet)-W_-(\tau-\delta\tau,\bullet)]}{\delta\tau}\right|\mathscr{F}_\tau\right]\!\!\right](\omega) \tag{6}$$

We name "the dual-progressively measurable process", or shortening "D-progressive" and also "the D-process", such a $\{\mathscr{P}_\tau\}$-progressive and $\{\mathscr{F}_\tau\}$-progressive $\hat{x}(\circ,\bullet)$ (3-4) instead of Nelson's (S2) and (S3)-process [2]. Of cause, $g\in\mathbb{V}_{\mathrm{M}}^4\otimes\mathbb{V}_{\mathrm{M}}^4$ is the metric in the Minkowski spacetime $(\mathbb{A}^4(\mathbb{V}_{\mathrm{M}}^4,g),\mathscr{B}(\mathbb{A}^4(\mathbb{V}_{\mathrm{M}}^4,g)),\mu)$ with its signature $g=\mathrm{diag}(+1,-1,-1,-1)$. The differential form of (3-4) is also employed:

Kinematics→

$$\boxed{d\hat{x}^\mu(\tau,\omega)=\mathcal{V}_\pm^\mu(\hat{x}(\tau,\omega))d\tau+\lambda\times dW_\pm^\mu(\tau,\omega)} \tag{7}$$

Key is the "Dual" progressively measurable stochastic process!!

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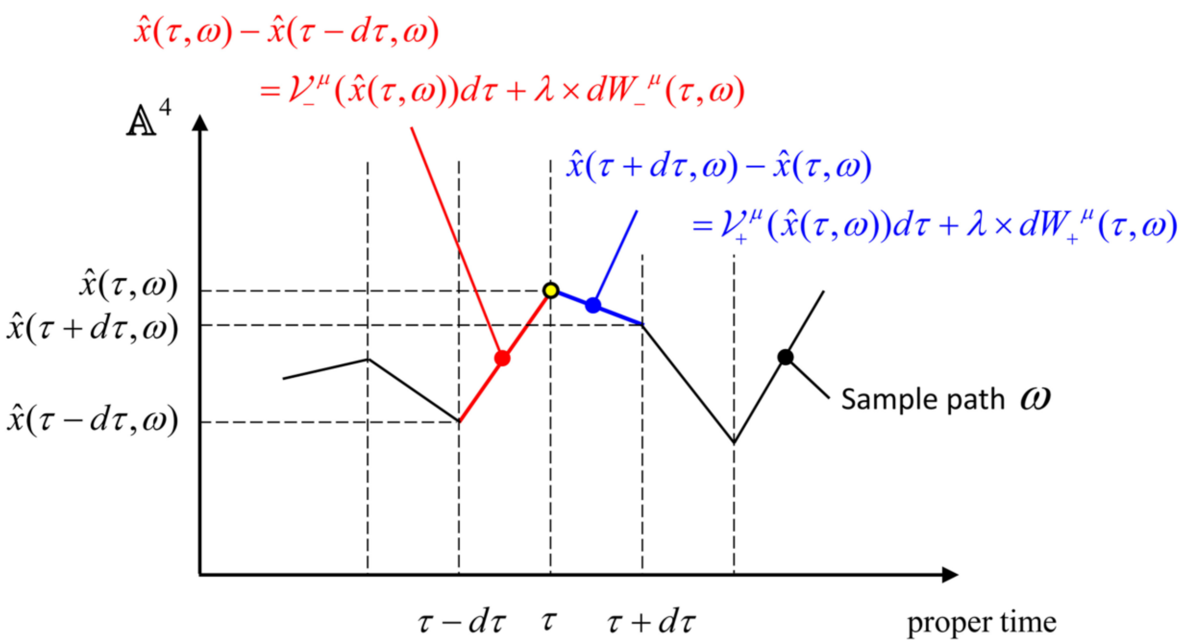
D-progressive: $d\hat{x}^{\mu}(\tau,\omega) = \mathcal{V}_{\pm}^{\mu}(\hat{x}(\tau,\omega))d\tau + \lambda \times dW_{\pm}^{\mu}(\tau,\omega)$



Itô rule: For each $\omega \in \Omega$,

$$d\tau \cdot d\tau = 0, \qquad d\tau \cdot dW_{\pm}^{\mu}(\tau,\omega) = 0,$$

$$dW_{\pm}^{\mu}(\tau,\omega) \cdot dW_{\pm}^{\nu}(\tau,\omega) = \mp g^{\mu\nu} d\tau$$

Stochastic Model vs Classical Model

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Stochastic version

Propagation of a scalar electron

$$\mathfrak{S}[\hat{x},\mathcal{V},\mathcal{V}^*,A]=\mathbb{E}\left[\!\!\left[\int_{\mathbb{R}} d\tau \frac{m_0}{2}\mathcal{V}^*_\alpha(\hat{x}(\tau,\bullet))\mathcal{V}^\alpha(\hat{x}(\tau,\bullet))\right]\!\!\right]$$

$$+\mathbb{E}\left[\!\!\left[-\int_{\mathbb{R}} d\tau\, e A_\alpha(\hat{x}(\tau,\bullet))\mathrm{Re}\{\mathcal{V}^\alpha(\hat{x}(\tau,\bullet))\}\right]\!\!\right]$$

Interaction term

$$+\int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)} d\mu(x)\frac{1}{4\mu_0 c}[F_{\alpha\beta}(x)+\delta f_{\alpha\beta}(x)]\cdot[F^{\alpha\beta}(x)+\delta f^{\alpha\beta}(x)]$$

Propagation of Field(s)

$$\begin{cases} m_0\mathfrak{D}_\tau\mathcal{V}^\mu(\hat{x}(\tau,\omega)) = -e\hat{\mathcal{V}}_\nu(\hat{x}(\tau,\omega))F^{\mu\nu}(\hat{x}(\tau,\omega)) \\ \mu_0^{-1}\times\partial_\mu[F^{\mu\nu}(x)+\delta f^{\mu\nu}(x)] = \mathbb{E}\left[\!\!\left[-ec\int_{\mathbb{R}} d\tau\,\mathrm{Re}\{\mathcal{V}^\nu(x)\}\delta^4(x-\hat{x}(\tau,\bullet))\right]\!\!\right] \end{cases}$$

Classical version

Propagation of a scalar electron

$$S_{\mathrm{classical}}[x,v,A]=\int_{\mathbb{R}} d\tau\frac{m_0}{2}v_\alpha(\tau)v^\alpha(\tau)$$

$$-\int_{\mathbb{R}} d\tau\, eA_\alpha(x(\tau))v^\alpha(\tau)$$

Interaction term

$$+\int_{\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)} d\mu(x)\frac{1}{4\mu_0 c}F_{\alpha\beta}(x)F^{\alpha\beta}(x)$$

Propagation of Field(s)

$$\begin{cases} m_0\dfrac{dv^\mu}{d\tau}=-eF^{\mu\nu}v_\nu \\ \mu_0^{-1}\times\partial_\mu F^{\mu\nu}=\left[-ec\int_{\mathbb{R}} d\tau\, v^\alpha(\tau)\delta^4(x-x(\tau))\right] \end{cases}$$

Definitions of their velocities: $\mathcal{V}\in\mathbb{V}^4_{\mathrm{M}}\oplus i\mathbb{V}^4_{\mathrm{M}}$ & $v\in\mathbb{V}^4_{\mathrm{M}}$

スカラー電子のDynamics(波動性)はKlein-Gordon方程式に従う

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K. Seto, arXiv: 1611.05861 (2016).

Theorem (Nelson):

Probability space: $\left(\Omega, D(\mathscr{P}), \mathscr{P}\right)$

$\omega \in \Omega$

Kinematics: (Itô integral)

$$d\hat{x}^{\mu}(\tau,\omega) = \mathcal{V}_{\pm}^{\mu}(\hat{x}(\tau,\omega))d\tau + \lambda \times dW_{\pm}^{\mu}(\tau,\omega)$$

Wiener process

Particle-wave duality

"particle"

"Wave"

Dynamics:

$$m_0 \mathfrak{D}_{\tau} \mathcal{V}^{\mu}(\hat{x}(\tau,\omega)) = -e\hat{\mathcal{V}}_{\nu}(\hat{x}(\tau,\omega))F^{\mu\nu}(\hat{x}(\tau,\omega))$$

Sub-eq:

$$\mathcal{V}^{\alpha}(x) = \frac{1}{m_0} \times [i\hbar\partial^{\alpha}\ln\phi(x) + eA^{\alpha}(x)]$$

$$= \frac{\mathcal{V}_{+}{}^{\alpha}(x) + \mathcal{V}_{-}{}^{\alpha}(x)}{2} - i\frac{\mathcal{V}_{+}{}^{\alpha}(x) - \mathcal{V}_{-}{}^{\alpha}(x)}{2}$$

= Klein-Gordon方程式

Ehrenfest

古典論

$$v^{\mu}(\tau) := \frac{d\mathbb{E}[\![\hat{x}^{\mu}(\tau,\bullet)]\!]}{d\tau}$$

$$m_0 \frac{dv^{\mu}}{d\tau}(\tau) = -ev_{\nu}F^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!])$$

Field(s)
= Maxwell equation

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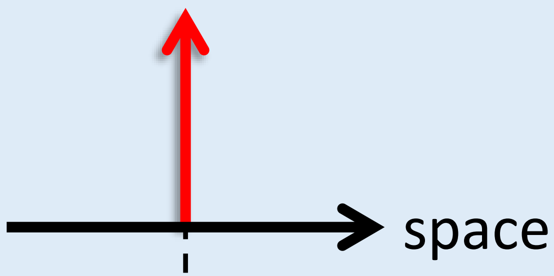
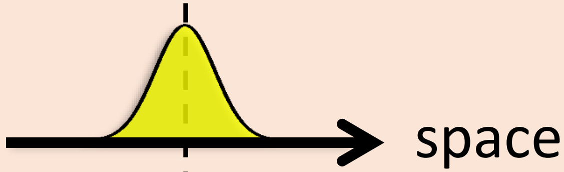
$$\partial_\mu [F^{\mu\nu}(x) + \delta f^{\mu\nu}(x)] = \mu_0 \times j^\mu_{\text{classical, Klein-Gordon, stochastic}}(x)$$



Classical model

$$j^\mu_{\text{classical}}(x) = -ec\int_{-\infty}^{\infty} d\tau\, w^\mu(\tau)\delta^4(x - x(\tau))$$

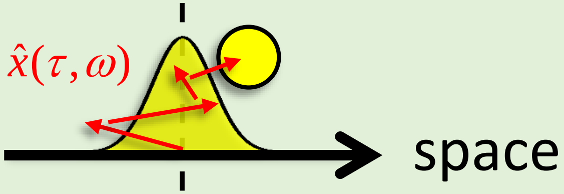


Scalar QED model

$$\mathfrak{D}_\nu = \partial_\nu - \frac{ie}{\hbar}A_\nu(x)$$

$$j^\mu_{\text{Klein-Gordon}}(x) = -\frac{ie\hbar}{2m_0} \times g^{\mu\nu}\left[\phi^*(x)\mathfrak{D}_\nu\phi(x) - \phi(x)\mathfrak{D}^*_\nu\phi^*(x)\right]$$

$$\parallel$$



Brownian motion model

stochastic process

$$j^\mu_{\text{stochastic}}(x) = \mathbb{E}\left[\!\left[-ec\int_{\mathbb{R}} d\tau\, \mathrm{Re}\{\mathcal{V}^\mu(x)\}\delta^4(x - \boxed{\hat{x}(\tau,\bullet)})\right]\!\right]$$

- K. Seto, arXiv: 1611.05458 (2016).

How to solve the Maxwell equation?
- Factor of "stochasticity"

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The theorem for the analysis of radiation from a stochastic scalar electron.

Theorem 4 (Factor of stochasticity). *In the Minkowski spacetime* $(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g), \mathscr{B}(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)), \mu)$, *consider an arbitrary* $\mathscr{B}(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g))/\mathscr{B}(\mathbb{R})$*-measurable and* C^∞*-local square integrable generalized-function* f *along the D-progressive* $\hat{x}(\circ, \bullet)$, *and it fulfills* $f(\mathbb{E}[\![\hat{x}(\tau, \bullet)]\!]) \neq 0$ *for each* $\tau \in \mathbb{R}$. *Then, a certain* C^∞*-function* $\Xi : \mathbb{R} \to \mathbb{R}$ *exists such that*

$$\boxed{\mathbb{E}[\![f(\hat{x}(\tau, \bullet))]\!] = \Xi(\tau) \times f(\mathbb{E}[\![\hat{x}(\tau, \bullet)]\!])}\,. \tag{29}$$

Definition 5 (Integral transformation). Consider **Theorem 4**, let $\hat{\mathcal{K}} f(t) := \mathbb{E}[\![f(\hat{x}(\tau, \bullet))]\!]$ be regarded as the integral transform with respect to its integral kernel $p(x, \tau) := \mathbb{E}[\![\delta^4(x - \hat{x}(\tau, \bullet))]\!]$ and $\hat{\mathcal{K}}' f(t) := \Xi(\tau) \times f(\mathbb{E}[\![\hat{x}(\tau, \bullet)]\!])$ by the kernel $p'(x, \tau) := \Xi(\tau) \times \delta^4(x - \mathbb{E}[\![\hat{x}(\tau, \bullet)]\!])$ for $x \in \mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}}, g)$. Hence, these two integral operators invoke the relation, $\hat{\mathcal{K}} = \hat{\mathcal{K}}'$.

Maxwell equation:

$$\begin{aligned}\mu_0^{-1} \times \partial_\mu [F^{\mu\nu}(x) + \delta f^{\mu\nu}(x)] &= \mathbb{E}\left[\!\!\left[-ec \int_{\mathbb{R}} d\tau \,\mathrm{Re}\left\{\mathcal{V}^\nu(x)\right\} \delta^4(x - \hat{x}(\tau, \bullet)) \right]\!\!\right] \\ &= -ec \int_{\mathbb{R}} d\tau \,\Xi(\tau) \mathrm{Re}\left\{\mathcal{V}^\nu(x)\right\} \delta^4(x - \mathbb{E}[\![\hat{x}(\tau, \bullet)]\!])\end{aligned}$$

Radiation reaction acting on a Brownian motion

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K. Seto, arXiv: 1611.05458 (2016).

Conclusion 9 (Radiation reaction). In the Minkowski spacetime $(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g),\mathscr{B}(\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)),\mu)$ with the probability space $(\Omega, D(\mathscr{P}),\mathscr{P})$, consider **Theorem 2**, namely, define the D-progressive $\hat{x}(\circ,\bullet) := \{\hat{x}(\tau,\omega)\in\mathbb{A}^4(\mathbb{V}^4_{\mathrm{M}},g)|\tau\in\mathbb{R},\omega\in\Omega\}$ as the stochastic kinematics of a scalar electron with the following dynamics of a stochastic scalar electron and a field characterized by $\mathcal{V}\in\mathbb{V}^4_{\mathrm{M}}\oplus i\mathbb{V}^4_{\mathrm{M}}$ and $\mathfrak{F}\in\mathbb{V}^4_{\mathrm{M}}\otimes\mathbb{V}^4_{\mathrm{M}}$:

$$m_0\mathfrak{D}_\tau\mathcal{V}^\mu(\hat{x}(\tau,\omega)) = -e\hat{\mathcal{V}}_\nu(\hat{x}(\tau,\omega))\left[F^{\mu\nu}_{\mathrm{ex}}(\hat{x}(\tau,\omega))+\mathfrak{F}^{\mu\nu}(\hat{x}(\tau,\omega))\right] \tag{74}$$

$$\partial_\mu\left[\pm\mathfrak{F}^{\mu\nu}(x)+\delta\mathfrak{f}^{\mu\nu}(x)\right] = \mu_0\times\mathbb{E}\left[\!\left[-ec\int_{\mathbb{R}}d\tau'\,\mathrm{Re}\{\mathcal{V}^\nu(x)\}\,\delta^4(x-\hat{x}(\tau',\bullet))\right]\!\right] \tag{75}$$

Where, $F_{\mathrm{ex}}\in\mathbb{V}^4_{\mathrm{M}}\otimes\mathbb{V}^4_{\mathrm{M}}$ satisfies $\partial_\mu F^{\mu\nu}_{\mathrm{ex}}=0$ and the dynamics of (74) is equivalent to the Klein-Gordon equation. For the retarded and advanced fields $\mathcal{F}_{(\pm)}=\pm\mathfrak{F}+\delta\mathfrak{f}\in\mathbb{V}^4_{\mathrm{M}}\otimes\mathbb{V}^4_{\mathrm{M}}$, $\mathfrak{F}=[\mathcal{F}_{(+)}-\mathcal{F}_{(-)}]/2$ and $\delta\mathfrak{f}\in\mathbb{V}^4_{\mathrm{M}}\otimes\mathbb{V}^4_{\mathrm{M}}$ represent the homogeneous solution of (75) such that $\partial_\mu\mathfrak{F}^{\mu\nu}=0$ and its singularity (72-73). Hence, the full dynamics of the radiating stochastic scalar electron is as follows:

Derivation: K. Seto, arXiv: 1611.05458 (2016).

$$\begin{aligned} m_0\mathfrak{D}_\tau\mathcal{V}^\mu(\hat{x}(\tau,\omega)) =& -e\hat{\mathcal{V}}_\nu(\hat{x}(\tau,\omega))F^{\mu\nu}_{\mathrm{ex}}(\hat{x}(\tau,\omega)) \\ &- e\mathcal{V}_\nu(\hat{x}(\tau,\omega))\left[\begin{array}{c}\mathfrak{F}^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]) \\ +\delta\hat{x}^\alpha(\tau,\omega)\cdot\partial_\alpha\mathfrak{F}^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!])\end{array}\right] + O\left(\overset{2}{\otimes}\delta\hat{x}(\tau,\omega)\right)\end{aligned} \tag{76}$$

This is the quantized equation of the LAD equation in classical dynamics,

$$m_0\frac{dv^\mu}{d\tau} = -ev_\nu F^{\mu\nu}_{ex} + \frac{m_0\tau_0}{c^2}\left(\frac{d^2v^\mu}{d\tau^2}v^\nu - \frac{d^2v^\nu}{d\tau^2}v^\mu\right)v_\nu\,. \tag{77}$$

Radiation reaction:
- Averaged trajectory

By Keita Seto (ELI-NP/IFIN-HH)

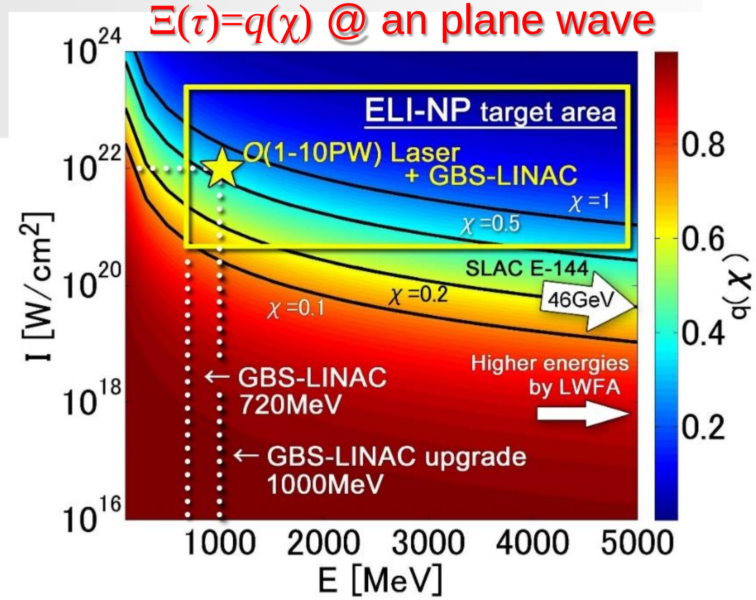
Average of a Brownian scalar electron with radiation reaction

$$m_0 \frac{d^2}{d\tau^2}\mathbb{E}[\![\hat{x}^\mu(\tau,\bullet)]\!] = -e\left[F_{\text{ex}}^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]) + \mathfrak{F}^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!])\right]\frac{d\mathbb{E}[\![\hat{x}_\nu(\tau,\bullet)]\!]}{d\tau} + O\left(\overset{2}{\otimes}\delta\hat{x}(\tau,\omega)\right)$$

$$\begin{cases} \mathfrak{F}^{\mu\nu}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]) = -\dfrac{m_0\tau_0\Xi(\tau)}{ec^2}\times\left[\dot{a}^\mu(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!])\cdot\dfrac{d\mathbb{E}[\![\hat{x}^\nu(\tau,\bullet)]\!]}{d\tau} - \dot{a}^\nu(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!])\cdot\dfrac{d\mathbb{E}[\![\hat{x}^\mu(\tau,\bullet)]\!]}{d\tau}\right] \\ \dot{a}(\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]) = \dfrac{d^3\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]}{d\tau^3} + \dfrac{3}{2}\dfrac{d\ln\Xi(\tau)}{d\tau}\cdot\dfrac{d^2\mathbb{E}[\![\hat{x}(\tau,\bullet)]\!]}{d\tau^2} \end{cases}$$

$$\frac{dW}{dt} = \Xi(\tau)\times\frac{dW_{\text{classical}}}{dt}$$

$\Xi(\tau)=q(\chi)$ @ an plane wave



LAD equation
("purely" classical physics)

$$m_0\frac{dv^\mu}{d\tau} = -eF_{\text{ex}}{}^{\mu\nu}v_\nu + \frac{m_0\tau_0}{c^2}\left(\frac{d^2v^\mu}{d\tau^2}v^\nu - \frac{d^2v^\nu}{d\tau^2}v^\mu\right)v_\nu$$

Summary:
Brownian motion for high-intensity field physics

By Keita Seto (ELI-NP/IFIN-HH)

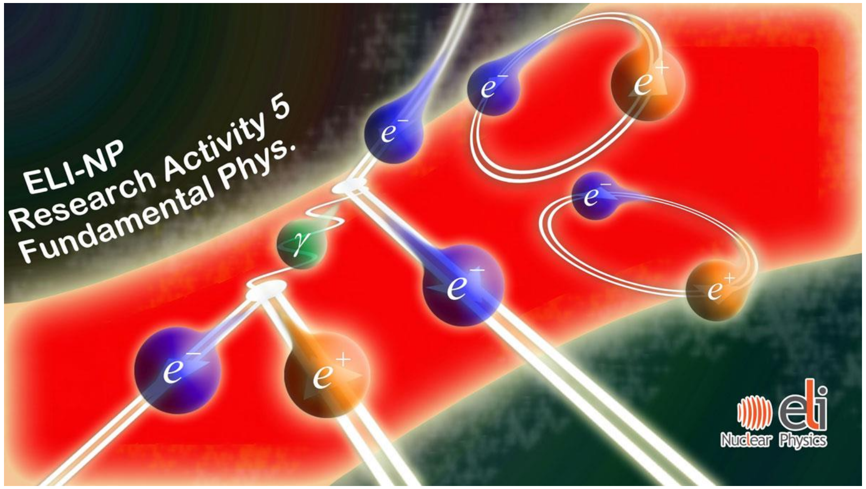
1) Nelson's stochastic quantization
+ Lorentz invariant
+ field generation

2) Radiation reaction on a Brownain motion

Future works

- Extension of mathematical model
- Numerical simulation
- Seeking the potential collaborators

& candidates of ELI-NP regular members



Thank you for your attention!!
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