Hydro-instabilities for Converging and Diverging System Governed by Power Law

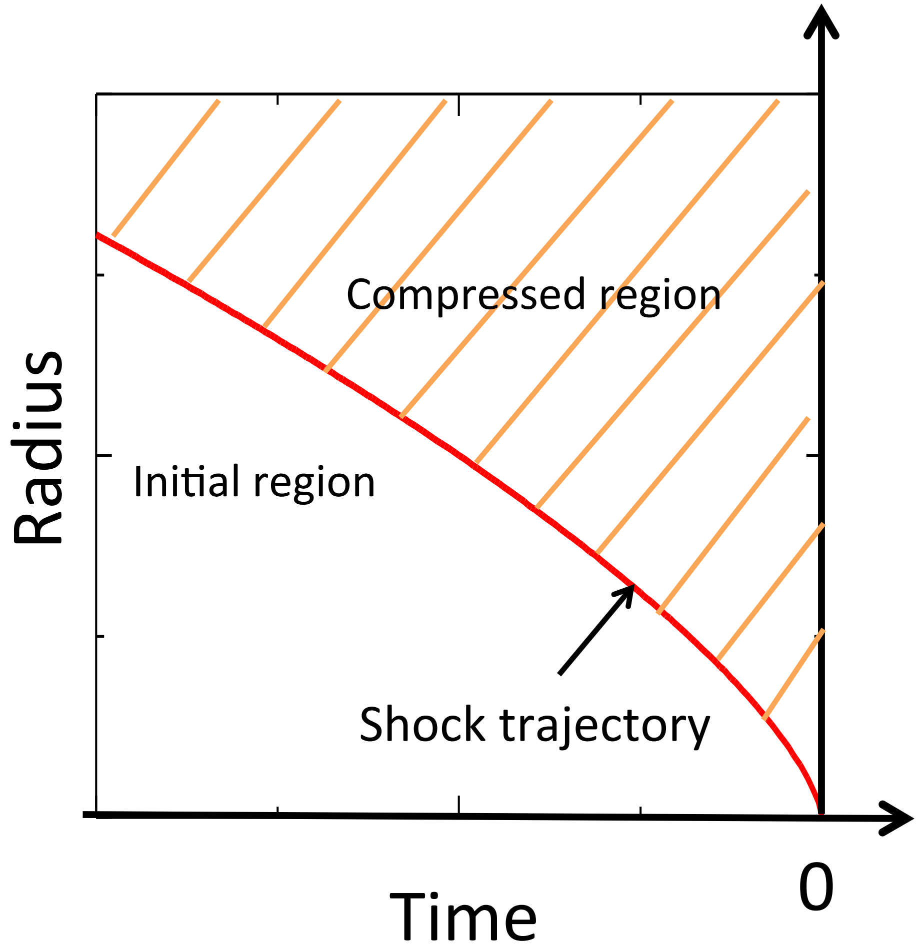
M. Murakami,[1]

A. L. Velikovich,[2] J. L. Giuliani,[2] B. D. Taylor,[3] S. T. Zalesak,[4]
S. Ohsako,[1] Y. Iwamoto[5]

[1]Institute of Laser Engineering, Osaka University, Japan
[2]Plasma Physics Division, Naval Research Laboratory, Washington, DC, USA
[3]Laboratory for Comp. Phys. & Fluid Dynamics, Naval Research Laboratory.
[4]Berkeley Research Associates, Beltsville, MD, USA
[5]Ehime University, Japan

- **Converging Shock**
- **Hyper-Spherical Converging & Diverging Shock**
- **Noh Problem**
- **RT Growth of Ablating Thin Foil**

Guderley's self-similar solution (1942)



Similarity Ansatz

$$R_S \propto |t|^{\alpha}$$

cf. $R_S \sim t^{2/5}$
Taylor-Sedov Explosion

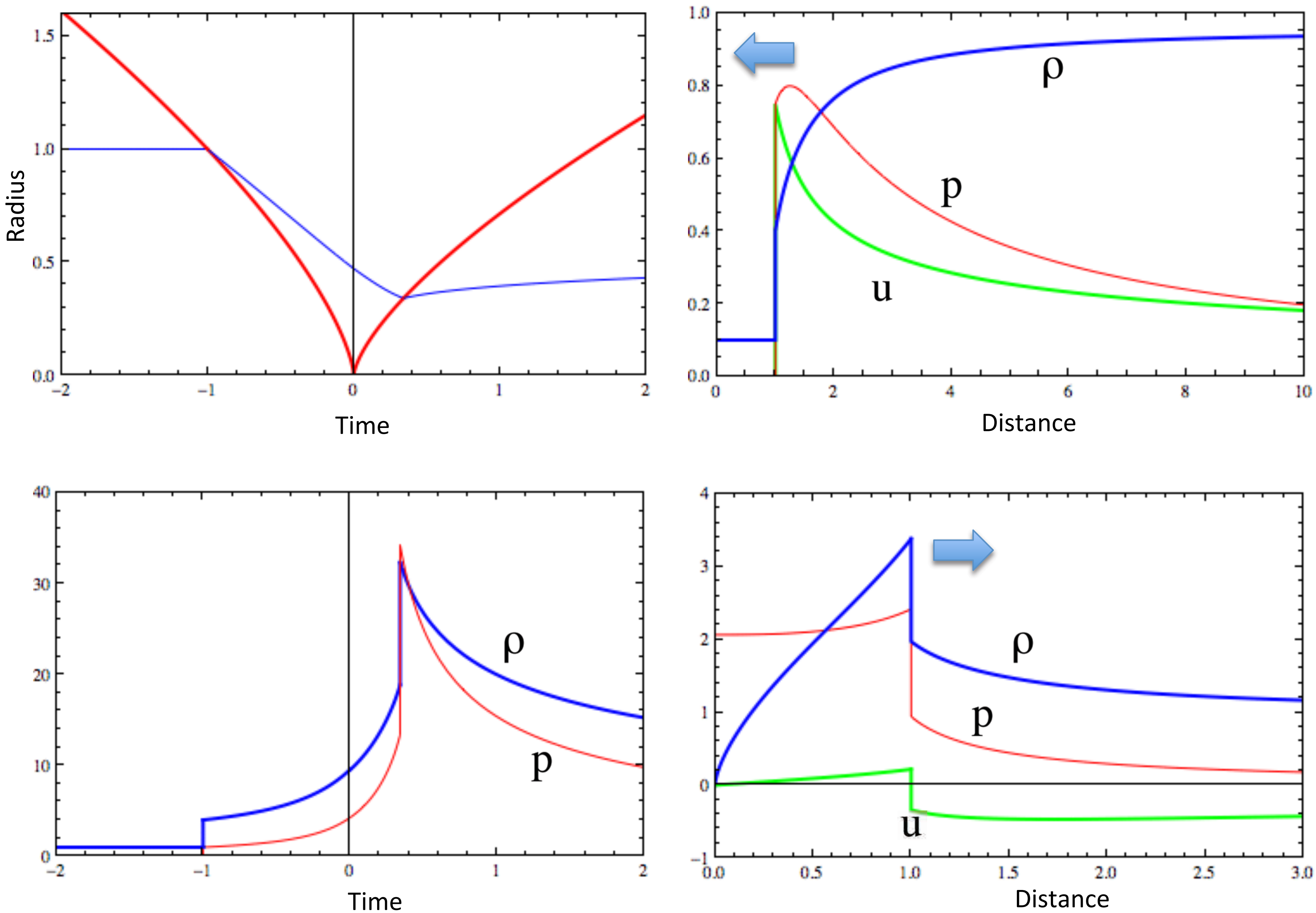
$$\xi \equiv \frac{r}{R_S} = \frac{r}{A|t|^{\alpha}}$$

Density $\quad \rho = \rho_0 G_0(\xi)$

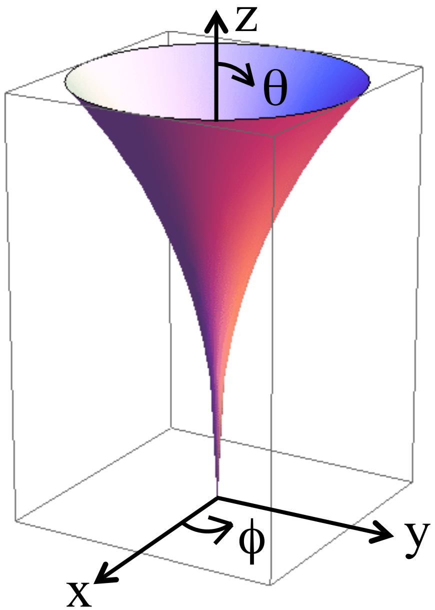
Velocity $\quad u = \frac{r}{t} V_0(\xi)$

Pressure $\quad p = \frac{r^2}{t^2} P_0(\xi)$

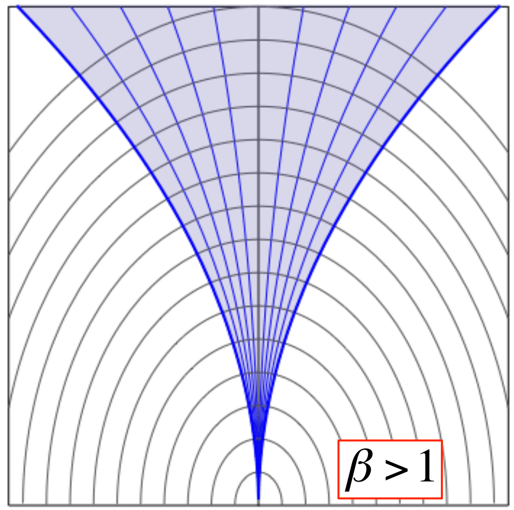
Time and spatial properties $\nu = 3$ Spherical case $\gamma = 5/3$



We propose new geometries corresponding to $\nu > 3$



(a) Trumpet target



$(x, y, z) \to (r, \theta, \phi)$

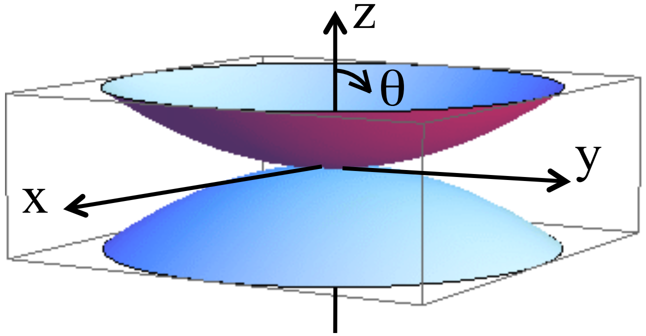
$$\sqrt{x^2 + y^2} = \tan\theta \cdot z^{\beta}$$

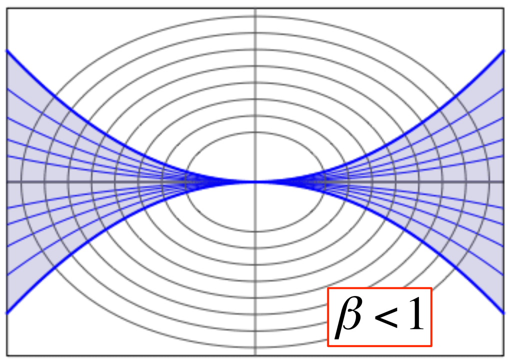
$$\beta(x^2 + y^2) + z^2 = r^2$$

$$y / x = \tan\phi$$

$\nu = 1 + 2\beta$

(b) Biconcave target

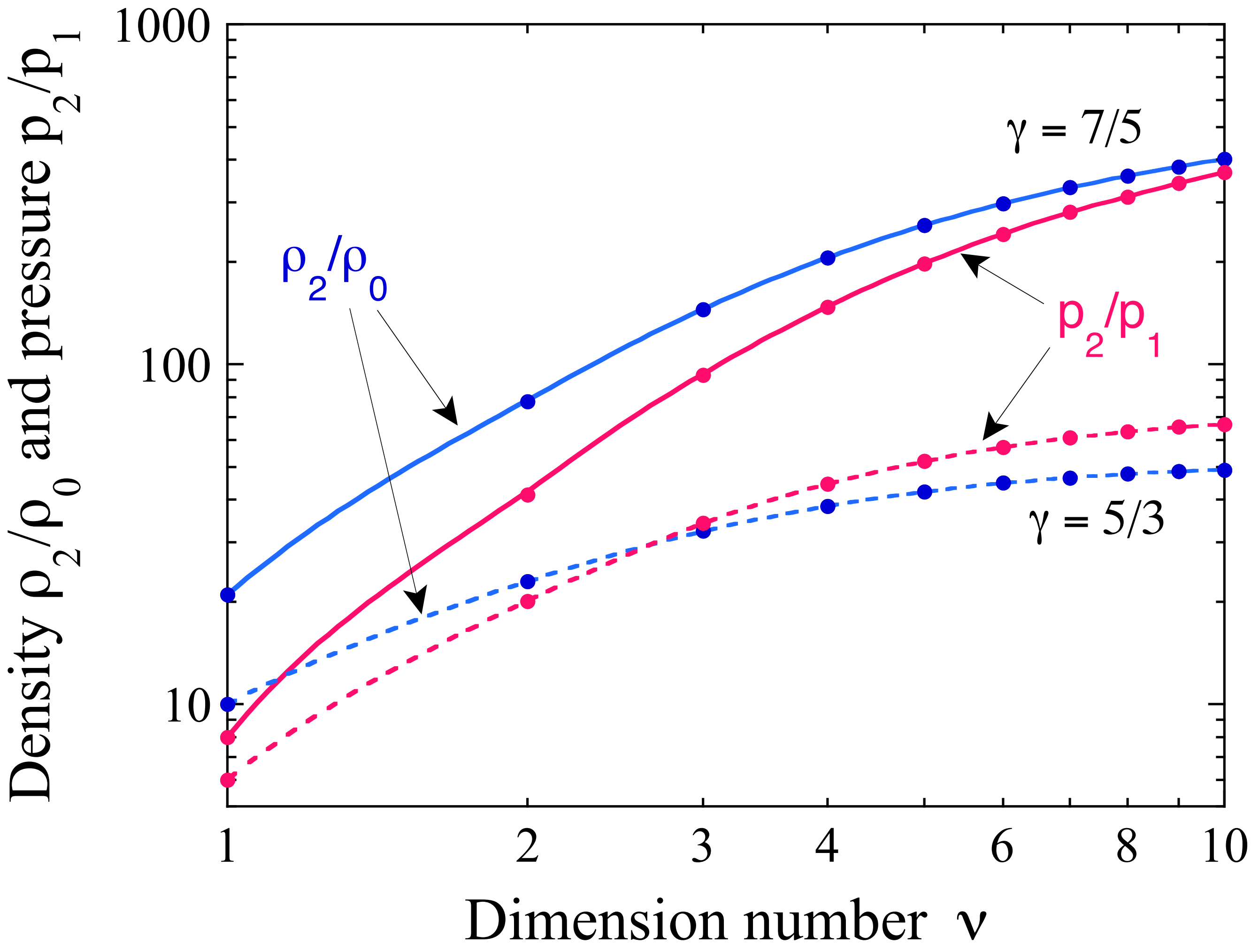




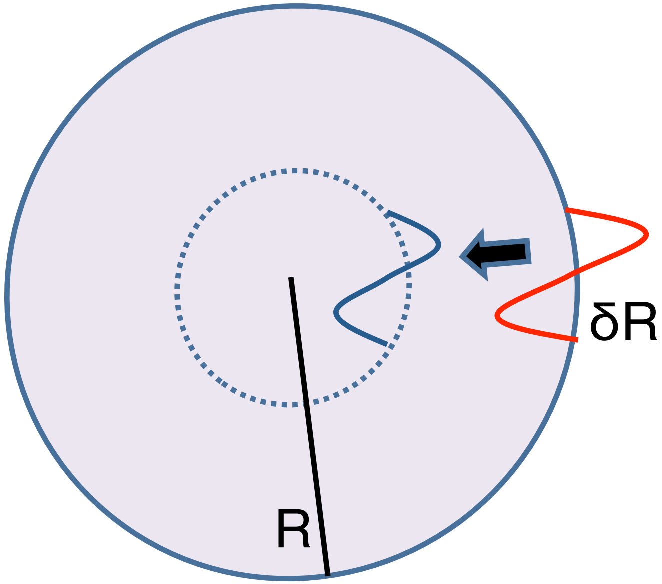
$\nu = 2 + \beta^{-1}$

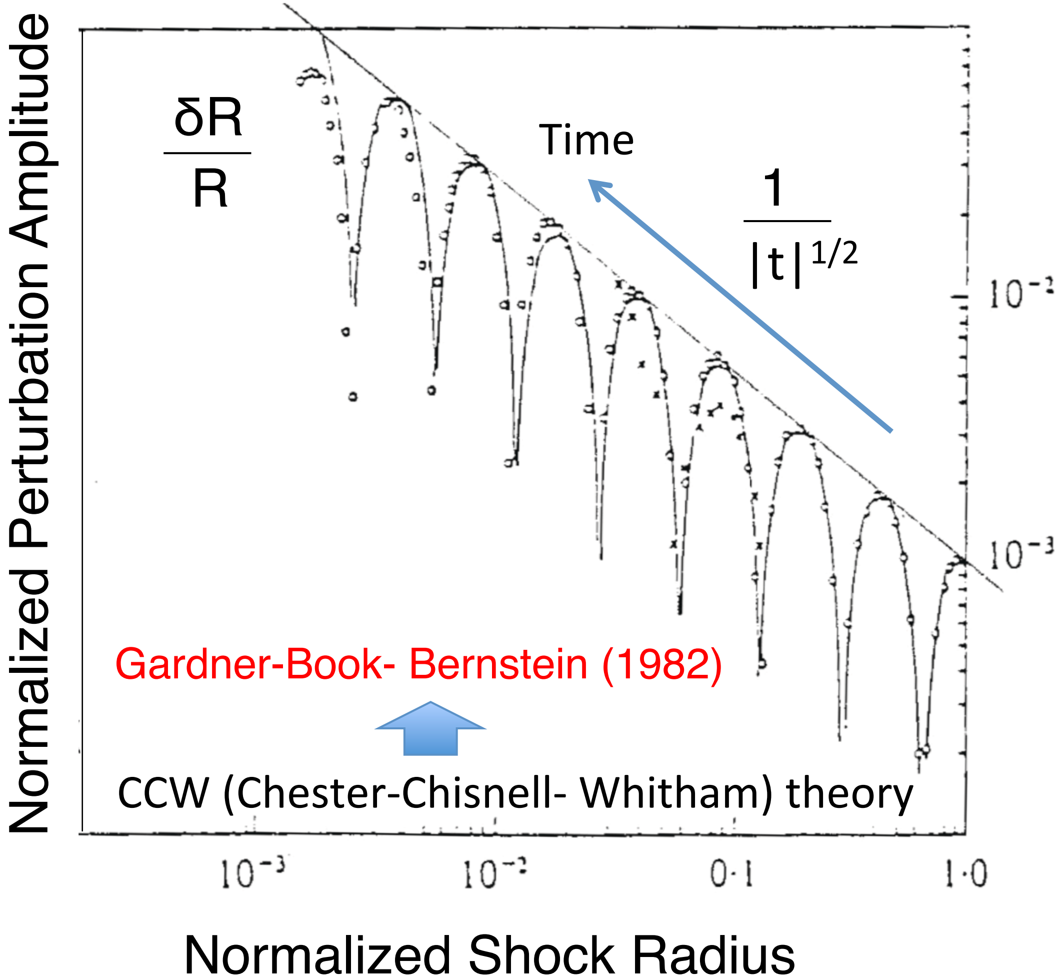
In the limit of θ→0, the shock propagation in the matter embedded in these targets is expected to reduc to one-dimensional problem.

After a single round-trip of shock, density compression rate amounts as high as 400 times the solid density with $\gamma = 7/5$.



Growth of Surface Perturbation of Converging shock





First order system for the perturbation

- Mass

$$\left(\frac{3}{\xi}+\frac{d}{d\xi}\right)(G_0V_1+G_1V_0)+\frac{\sigma G_1+G_0D_1}{\xi}-\alpha G_1'=0$$

- Momentum (azimuthal)

$$(2V_0+\sigma-1)V_1+\xi((V_0-\alpha)V_1)'=\frac{G_1(\xi^2P_0)'-G_0(\xi^2P_1)'}{\xi G_0{}^2}$$

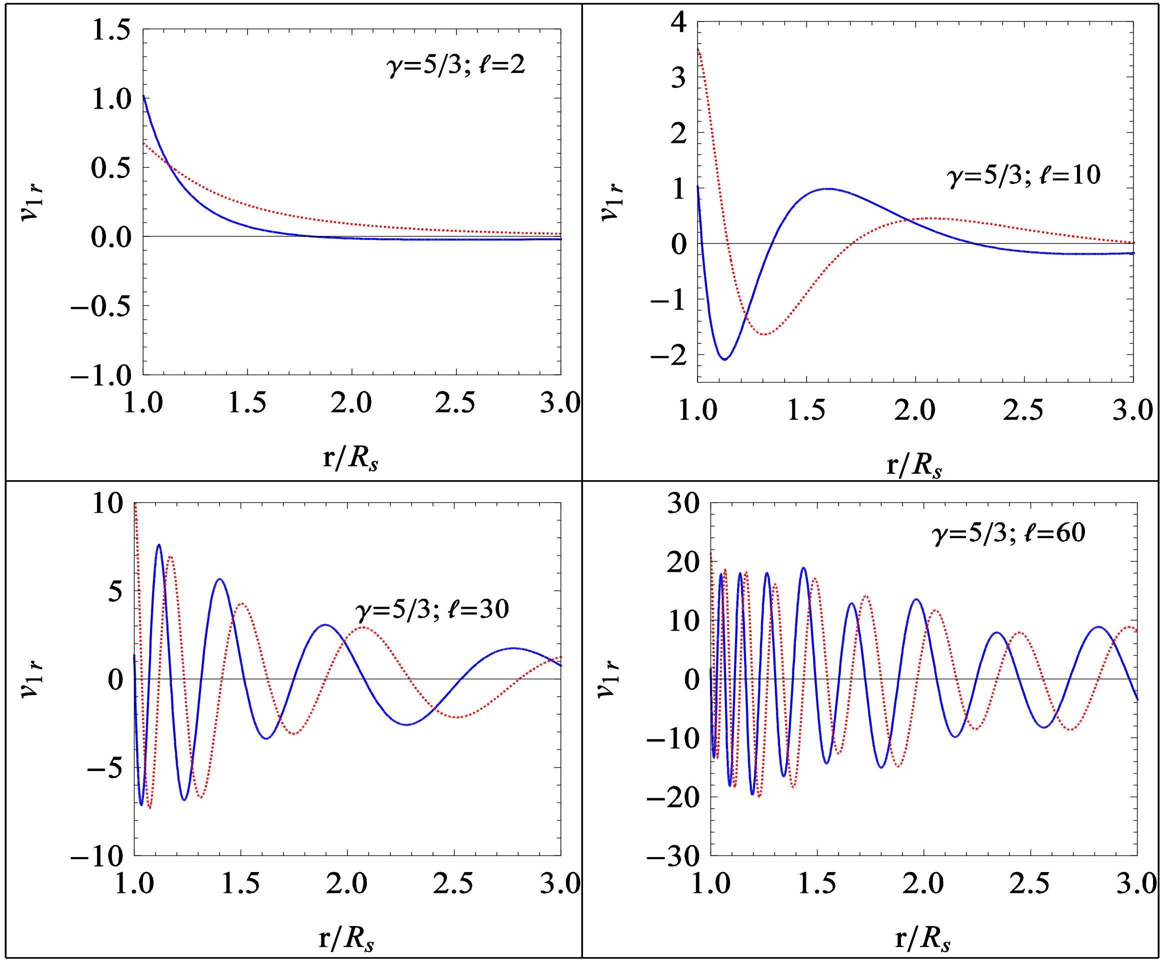
- Momentum (perpendicular)

$$(2V_0+\sigma-1)D_1+\xi(V_0-\alpha)D_1'=l(l+1)\frac{P_1}{G_0}$$

- Energy

$$\left(\frac{\sigma}{\xi}+(V_0-\alpha)\frac{d}{d\xi}\right)\left(\frac{P_1}{P_0}-\gamma\frac{G_1}{G_0}\right)+V_1\ln(\xi^2P_0G_0{}^{-\gamma})'=0$$

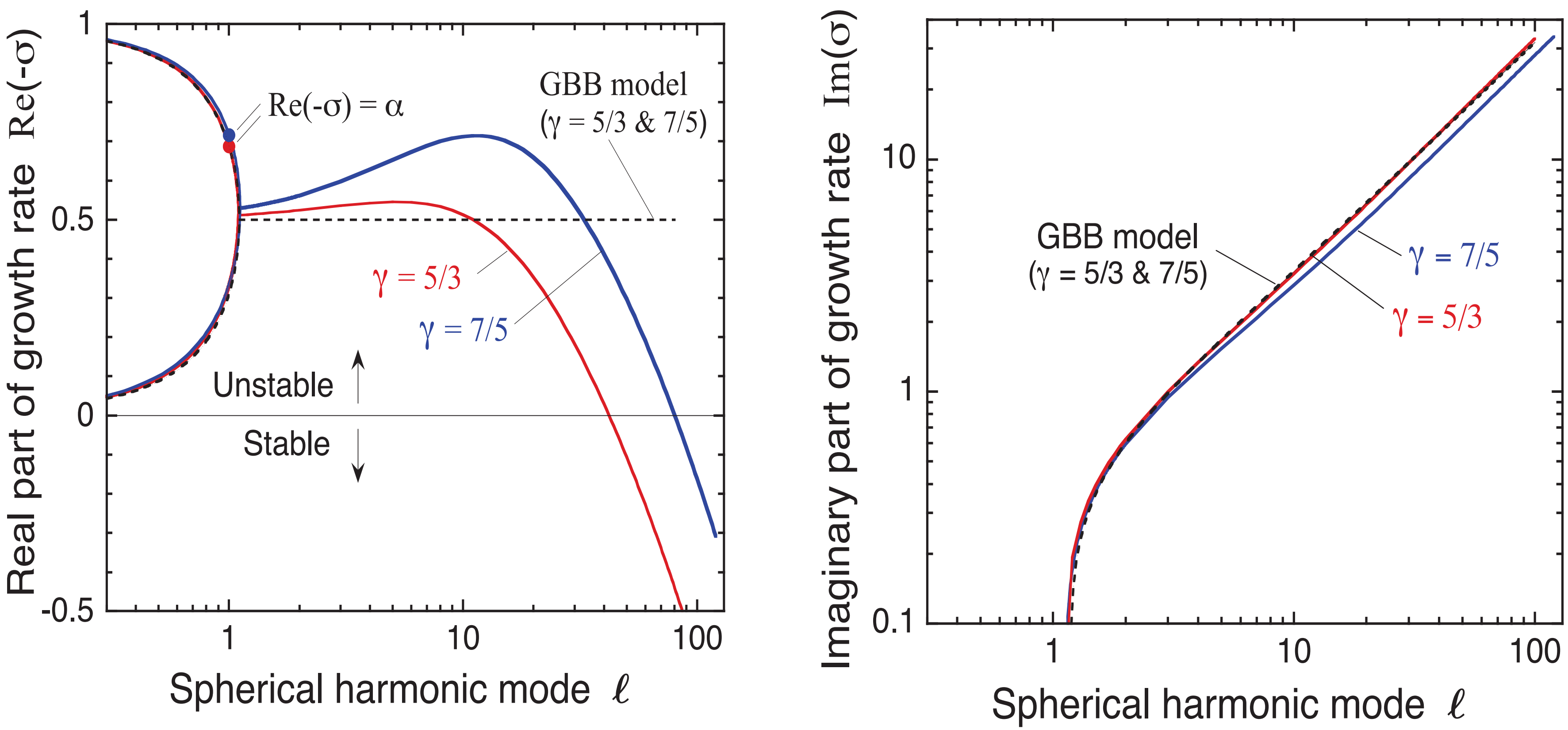
Eigenfunctions for spherical converging shocks



Perturbed radial velocity (imaginary part in blue, real part in red)

Cut-off modes exist, over which converging shock waves are stabilized even without conduction and viscosity.

M. Murakami *et al*, Phys. Plasmas **22**, 072703 (2015)



The classic spherical Noh solution[1] (reminder)

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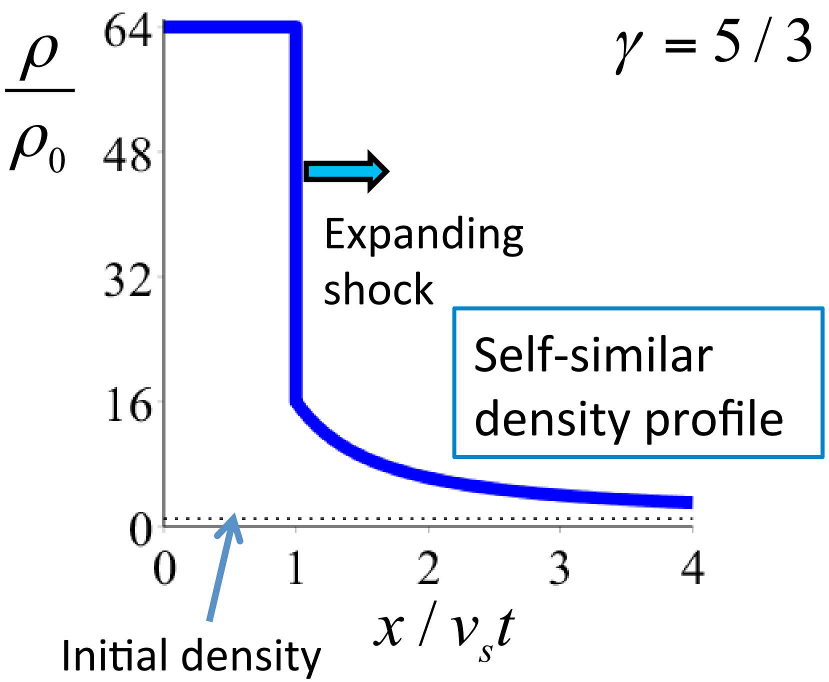
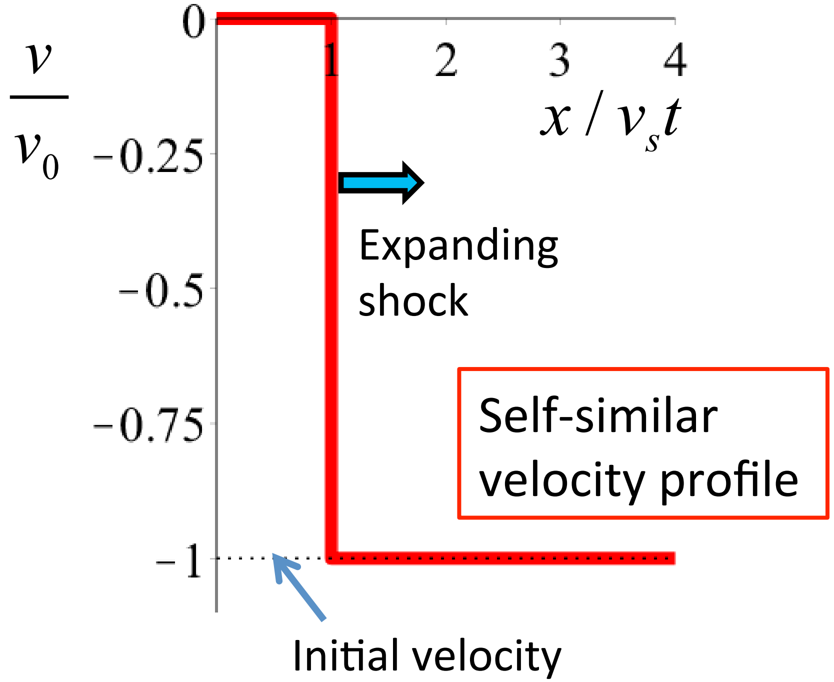
Initial density and velocity profiles are flat and spherically symmetric:

$$\rho = \rho_0 \text{ and } v = -v_0$$

After the shock reflection from the center, the density is increased 16× adiabatically before the shock front and 4× in the shock wave, total 64×





[1]W. F. Noh, J. Comput. Phys. **72**, 78 (1987).

Expanding-shock flows in cylindrical geometry: precursors and neutron production at stagnation

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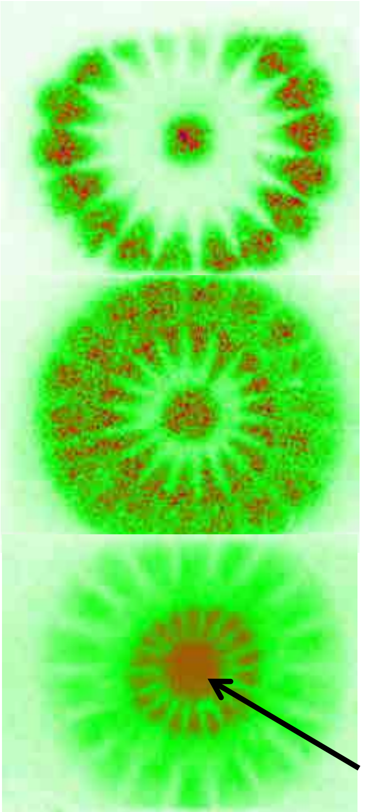
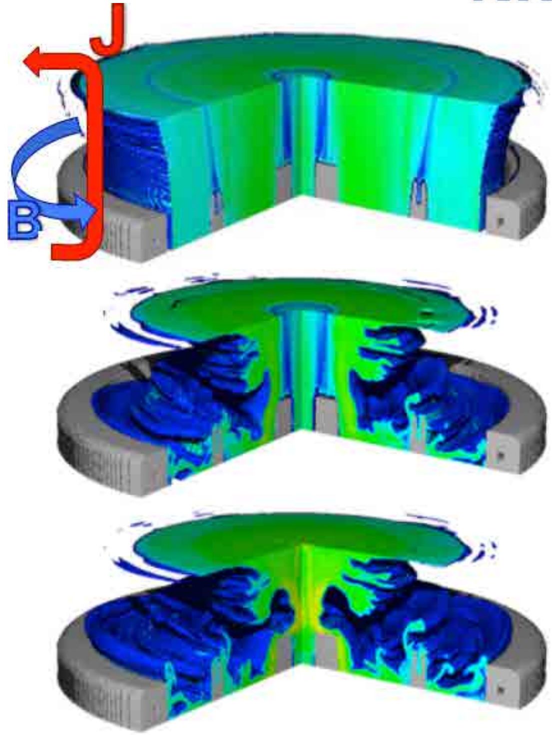


Precursor column formation observed end-on in M. Cuneo's experiment on Z with a 20 mm tungsten cylindrical wire array. [1]There is evidence that the peak of a Z-pinch x-ray emission power[2] and DD neutron production[3] up to 4×10^{13} is achieved in Noh-like stagnation via a shock wave. Generalization of the classic Noh solution might be needed for analysis.[4]

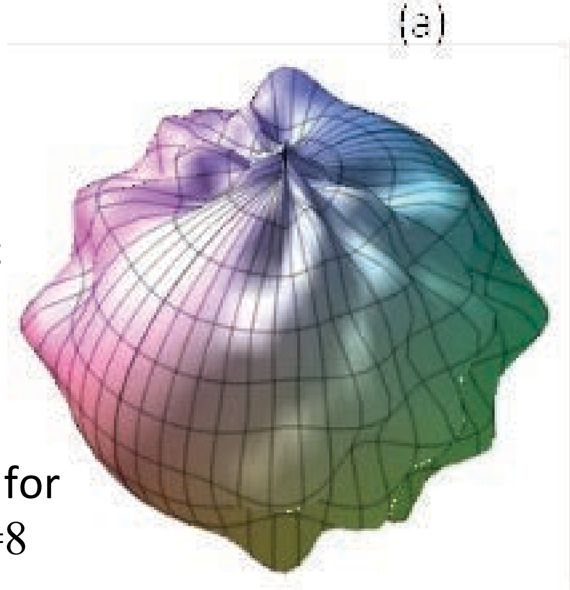
Precursor column



Deuterium gas puff Z-pinch: simulations C. Jennings, experiment on Z P. Knapp, Sandia

[1]S. V. Lebedev *et al*., Wire Array Workshop, Colorado Springs, CO, May 2003.
[2]Y. Maron *et al*., PRL **111**, 035001 (2013).
[3]C. A. Coverdale *et al*., Phys. Plasmas **14**, 022706 (2007); *ibid*., **14**, 056309 (2007).
[4]E. P. Yu *et al*., Phys. Plasmas **21**, 082703 (2014); **this conf. invited talk UI3.00002, Thursday 2:30 pm.**

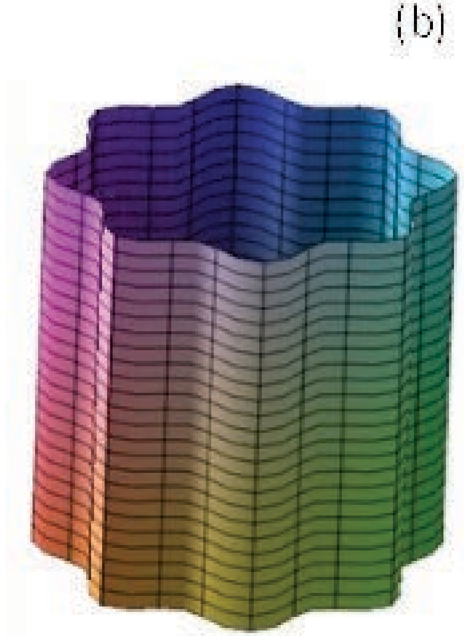
Perturbations of the classic Noh solutions



General spherical harmonic mode

Illustrated for $l = 12$, $m = 8$

$$r_s(\theta,\phi,t) = v_s t\left[1+\varepsilon\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_{l,m}}\eta_1^{l,m}Y_l^m(\theta,\phi)\right]$$



Filamentation cylindrical mode: no z dependence

Illustrated for $m = 8$

$$r_s(\phi,t) = v_s t\left[1+\varepsilon\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_m}\eta_1^{m}\exp(im\phi)\right]$$

Dimensionless eigenvalues σ have to be power indices of time

Separation of variables and normalized perturbation amplitudes[1]

Spherical | Cylindrical

$$\rho(r,\theta,\phi,t) = \rho_s\left[1+\varepsilon\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_{l,m}} G_1^{l,m}(\xi)Y_l^m(\theta,\phi)\right] \qquad \rho(r,\phi,t) = \rho_s\left[1+\varepsilon\sum_{m}\left(\frac{t}{t_0}\right)^{\sigma_{m}} G_1^{m}(\xi)\exp(im\phi)\right] \quad (1)$$

$$p(r,\theta,\phi,t) = p_s\left[1+\varepsilon\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_{l,m}} P_1^{l,m}(\xi)Y_l^m(\theta,\phi)\right] \qquad p(r,\phi,t) = p_s\left[1+\varepsilon\sum_{m}\left(\frac{t}{t_0}\right)^{\sigma_{m}} P_1^{m}(\xi)\exp(im\phi)\right] \quad (2)$$

$$v_r(r,\theta,\phi,t) = \varepsilon v_s\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_{l,m}} V_{r1}^{l,m}(\xi)Y_l^m(\theta,\phi) \qquad v_r(r,\phi,t) = \varepsilon v_s\sum_{m}\left(\frac{t}{t_0}\right)^{\sigma_{m}} V_{r1}^{m}(\xi)\exp(im\phi) \quad (3)$$

$$r\nabla_\perp\cdot\mathbf{v}_\perp(r,\theta,\phi,t) = \varepsilon v_s\sum_{l,m}\left(\frac{t}{t_0}\right)^{\sigma_{l,m}} D_1^{l,m}(\xi)Y_l^m(\theta,\phi) \qquad r\nabla_\perp\cdot\mathbf{v}_\perp(r,\phi,t) = \varepsilon v_s\sum_{m}\left(\frac{t}{t_0}\right)^{\sigma_{m}} D_1^{m}(\xi)\exp(im\phi) \quad (4)$$

Dimensionless perturbation amplitudes of density G_1 (1), pressure P_1 (2), radial velocity V_{r1} (3) and transverse divergence of transverse velocity D_1 (4).

Self-similar coordinate $\xi = r/(v_s t)$.

[1]M. Murakami, J. Sanz, and Y. Iwamoto, Phys. Plasmas **22**, 072703 (2015).

Dispersion equations and the eigenfunctions

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Dispersion equations

Spherical

$$\left[(\gamma-1)\sigma^2+(3\gamma-7)\sigma-(\gamma+1)l(l+1)+2\gamma-10\right]$$
$$\times {}_2F_1\left(\frac{l-\sigma}{2},\ \frac{l-\sigma+1}{2};\ l+\frac{3}{2};\ M_2^2\right)-2\left[(\gamma-1)\sigma+\gamma-3\right]$$
$$\times(\sigma+l+2)\,{}_2F_1\left(\frac{l-\sigma-1}{2},\ \frac{l-\sigma}{2};\ l+\frac{3}{2};\ M_2^2\right)=0\ .$$

Cylindrical

$$\left[(\gamma-1)\sigma^2+2(\gamma-2)\sigma-(\gamma+1)m^2+\gamma-3\right]$$
$$\times {}_2F_1\left(\frac{m-\sigma}{2},\ \frac{m+1-\sigma}{2};\ m+1;\ M_2^2\right)-2(\sigma+m+1)$$
$$\times\left[(\gamma-1)\sigma+\gamma-2\right]{}_2F_1\left(\frac{m-\sigma-1}{2},\ \frac{m-\sigma}{2};\ m+1;\ M_2^2\right)=0\ .$$

Pressure eigenfunctions

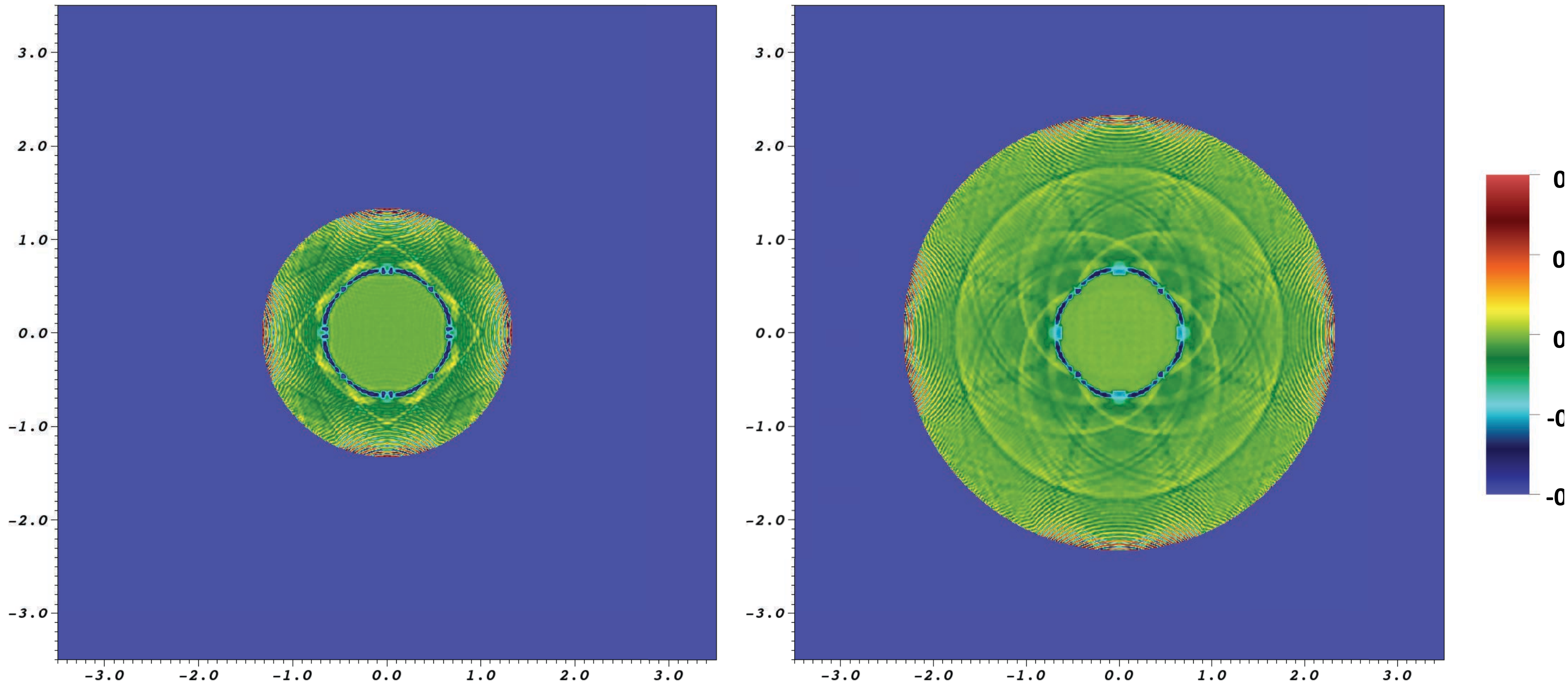
$$M_2=\sqrt{\frac{\gamma-1}{2\gamma}}$$

$$P_1=\frac{2\left[(\gamma-1)\sigma+\gamma-3\right]}{(\gamma+1)}\xi^l\times\frac{{}_2F_1\left(\frac{l-\sigma}{2},\ \frac{l-\sigma+1}{2};\ l+\frac{3}{2};\ M_2^2\xi^2\right)}{{}_2F_1\left(\frac{l-\sigma}{2},\ \frac{l-\sigma+1}{2};\ l+\frac{3}{2};\ M_2^2\right)}$$

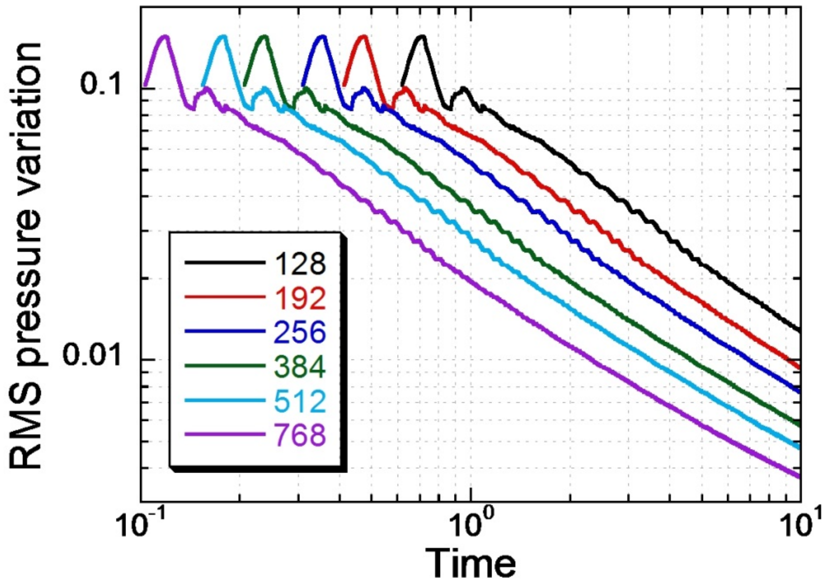
Sonic perturbations only

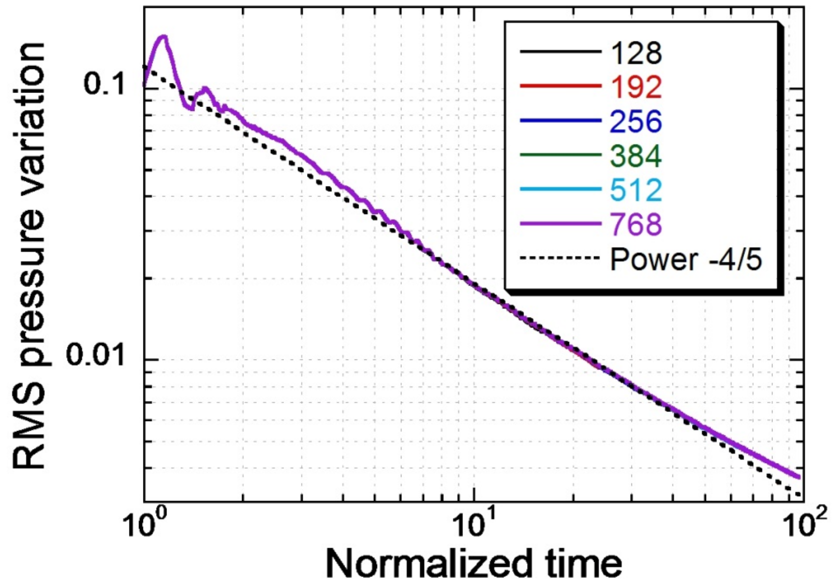
$$P_1=\frac{2\left[(\gamma-1)\sigma+\gamma-2\right]}{(\gamma+1)}\xi^m\times\frac{{}_2F_1\left(\frac{m-\sigma}{2},\ \frac{m-\sigma+1}{2};\ m+1;\ M_2^2\xi^2\right)}{{}_2F_1\left(\frac{m-\sigma}{2},\ \frac{m-\sigma+1}{2};\ m+1;\ M_2^2\right)}$$

Density perturbation map generated in a numerical solution of the cylindrical 2D Noh problem (cont.)



Numerical pressure variation for spherical 3D Noh decays with time as ~$t^{-4/5}$, slower than any of the eigenmodes



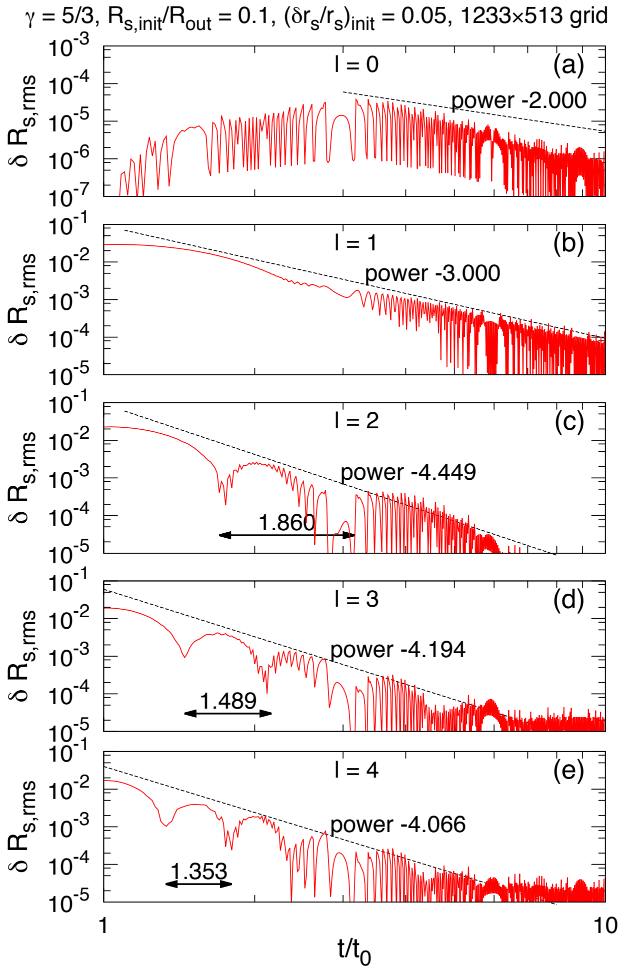


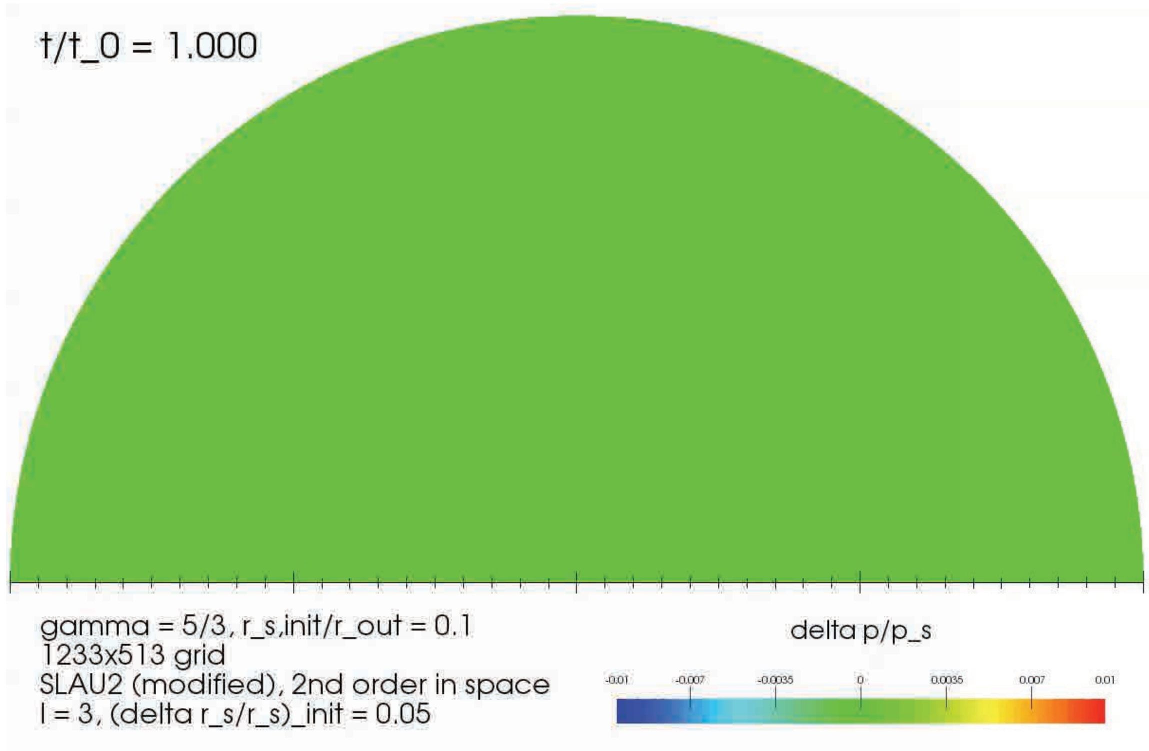
$$\left\langle \frac{\delta p}{p_s} \right\rangle_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_i}{p_s}-1\right)^2}$$

Sum over all cells behind the shock front.

With the time normalized as t/t_0 the time histories of δp for all resolutions collapse onto the same line ~$t^{-4/5}$

Spherical coordinate system





いまさらRT不安定性？

Self-consistent growth rate of the Rayleigh–Taylor instability in an ablatively accelerating plasma

H. Takabe and K. Mima
Institute of Laser Engineering, Osaka University, Suita, Osaka 565, Japan

L. Montierth and R. L. Morse
Department of Nuclear and Energy Engineering, University of Arizona, Tucson, Arizona 85721

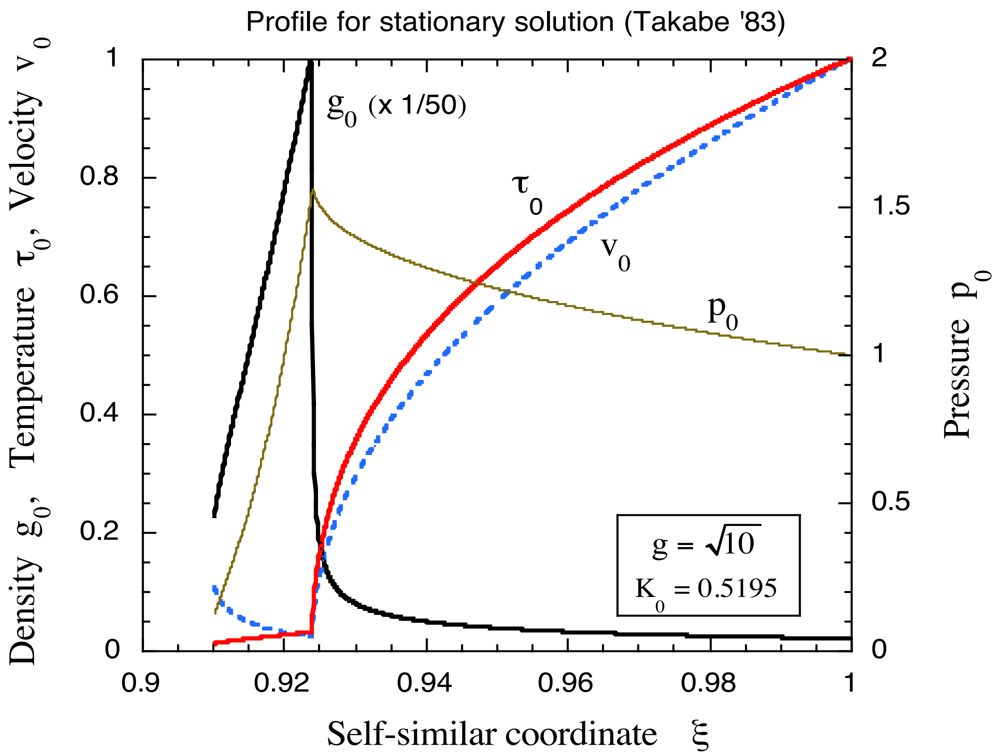
(Received 30 April 1984; accepted 3 September 1985)

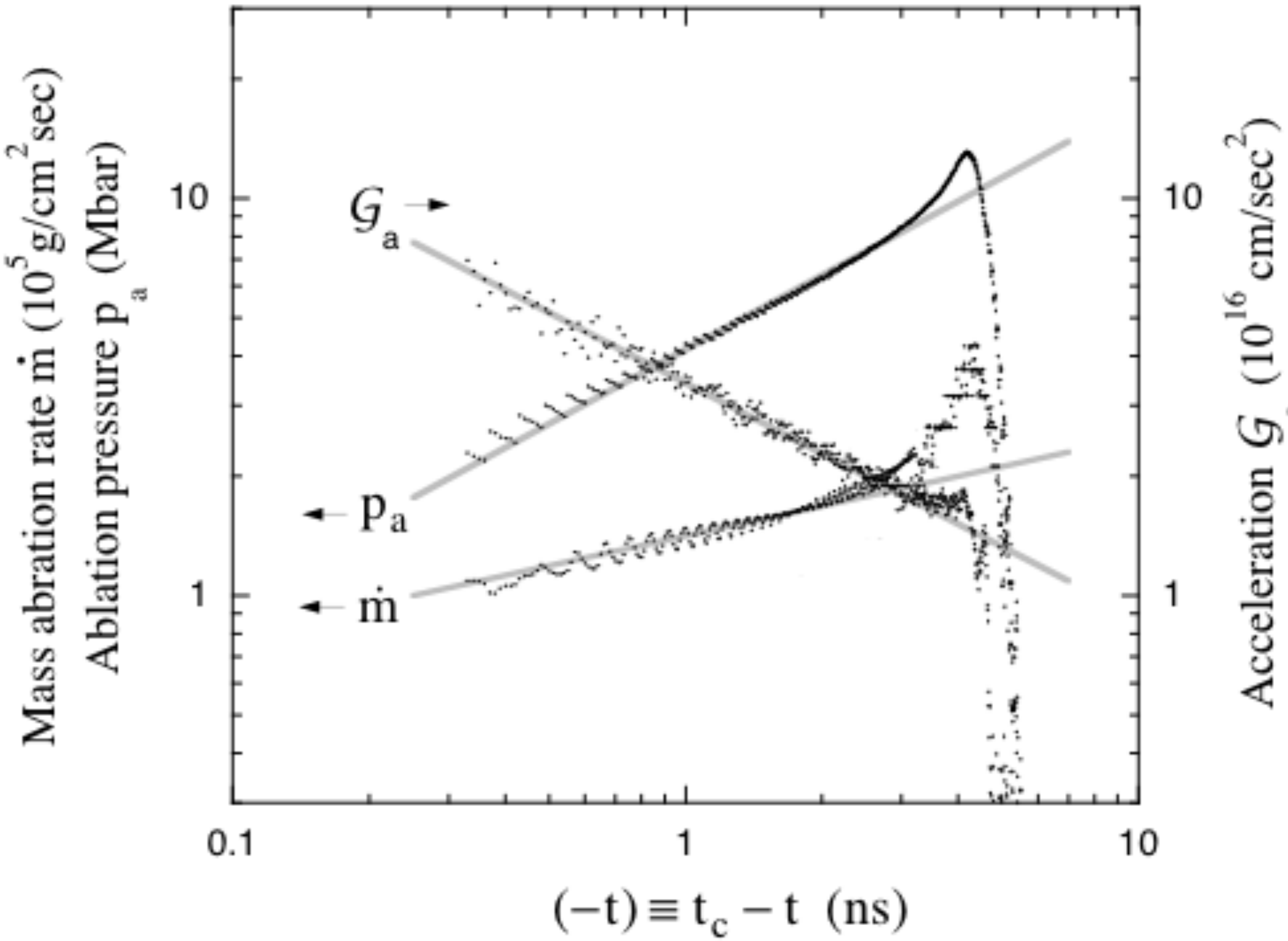
The linear stability of an ablating plasma is investigated as an eigenvalue problem by assuming the plasma to be at the stationary state. For various structures of the ablating plasma, the growth rate is found to be expressed well in the form $\gamma = \alpha\sqrt{kg} - \beta k v_a$, where $\alpha = 0.9$, $\beta \cong 3$–4, and v_a is the flow velocity across the ablation front, and is found to agree well with recent two-dimensional simulations in a classical transport regime. Short-wavelength lasers inducing enhanced mass ablation are suggested to be advantageous to stable implosion because of the ablative stabilization.

$$\cancel{\frac{\partial}{\partial t}}\rho + \nabla\cdot(\rho\mathbf{v}) = 0,$$

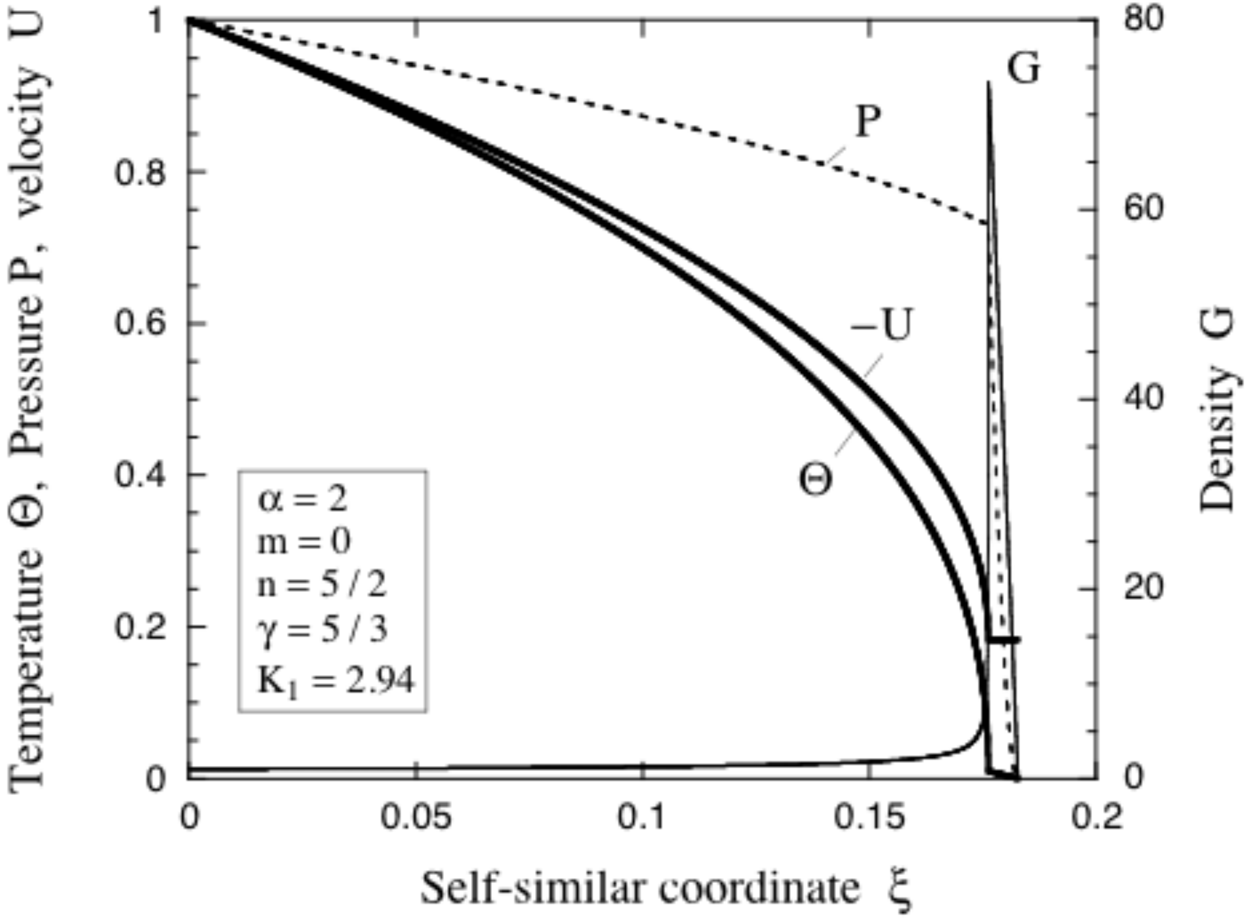
$$\rho\left(\cancel{\frac{\partial}{\partial t}}\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla P,$$

$$\rho C_v\left(\cancel{\frac{\partial}{\partial t}}T + \mathbf{v}\cdot\nabla T\right) = -P\nabla\cdot\mathbf{v} + \nabla\cdot(K_e\nabla T),$$

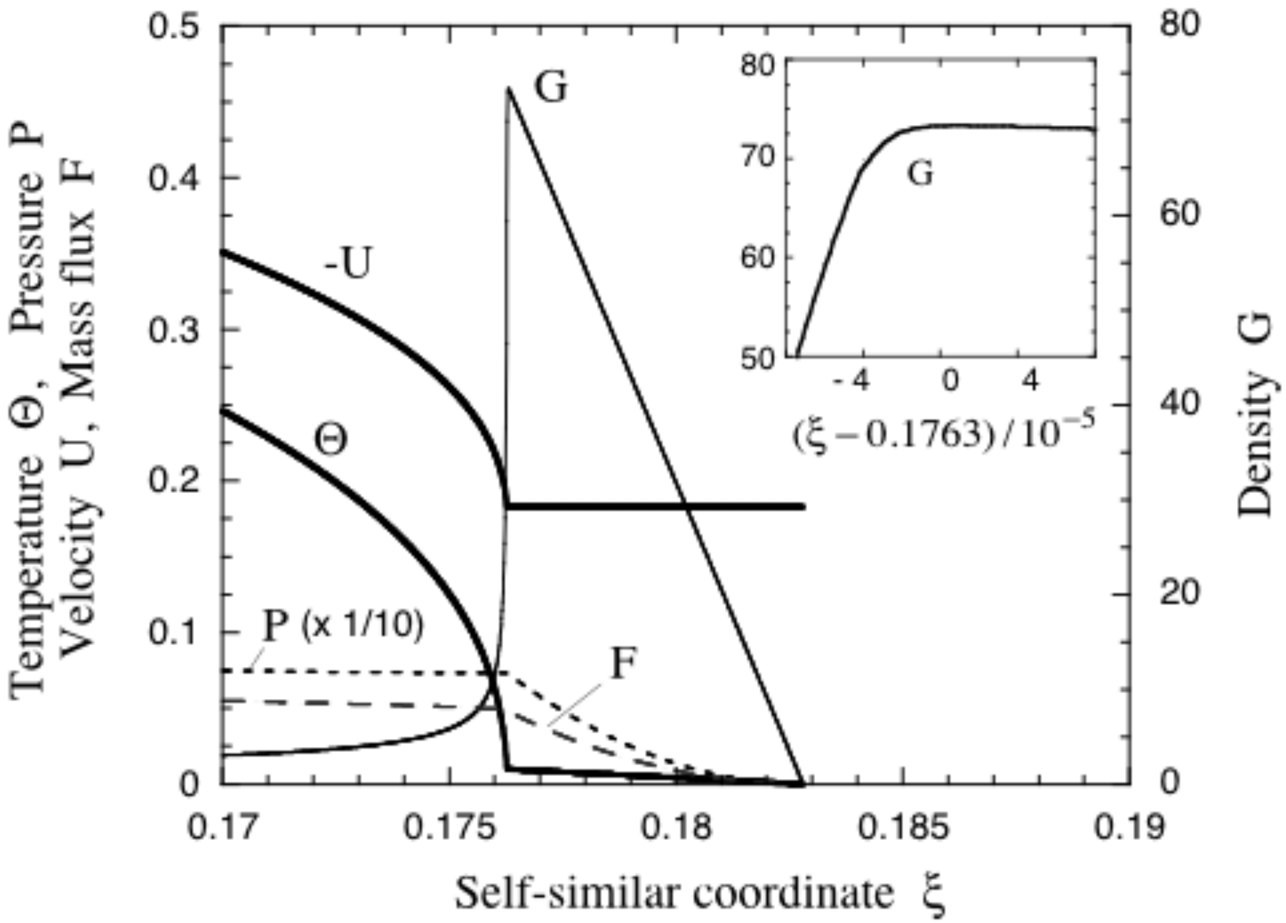




Mass abration rate ṁ (10^5 g/cm^{2}sec)
Ablation pressure p_a (Mbar)
Acceleration $\mathcal{G}_a$ (10^{16} cm/sec^2)
$\mathcal{G}_a$
p_a
ṁ
10
1
0.1
1
10
$(-t) \equiv t_c - t$ (ns)



Temperature Θ, Pressure P, velocity U
Density G
Self-similar coordinate ξ
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0
20
40
60
80
P
G
−U
Θ
α = 2
m = 0
n = 5 / 2
γ = 5 / 3
K1 = 2.94



Temperature Θ, Pressure P
Velocity U, Mass flux F
Density G
Self-similar coordinate ξ
-U
Θ
G
P (x 1/10)
F
0.5
0.4
0.3
0.2
0.1
0
0.17
0.175
0.18
0.185
0.19
80
60
40
20
0
(ξ − 0.1763) / 10^−5
- 4
4
70
50

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Conclusions

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- An analytic solution has been obtained for the small-amplitude perturbation analysis of the classic Noh problem
  - General (l, m) modes for the spherical geometry
  - Filamentation $(k = 0, m)$ modes for the cylindrical geometry
  - Dispersion equations and the eigenfunctions are all given by explicit analytic formulas
- All the perturbation modes decay with time as powers of time, indicating stability of the Noh solution
  - Oscillatory decay for most eigenmodes, monotonic for some
- Use of the new solutions for 2D/3D code verification is possible but challenging because the eigenfunctions might decay faster than the numerical noise

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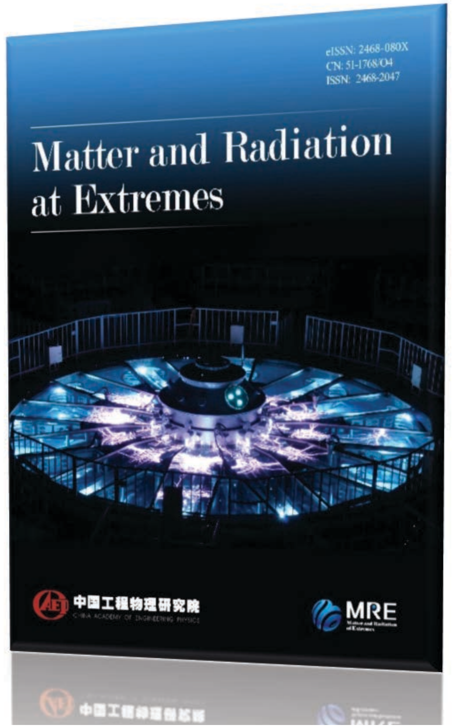

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