確率・統計 B

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学籍番号　　　　　　　　　　氏名

問題 $N(\mu, \sigma^2)$ のフィッシャー情報量行列を求めよ.

$$f(x;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\Big\{-\frac{1}{2\sigma^2}(x-\mu)^2\Big\}$$
$$\log f(x;\mu,\sigma^2)=-\frac{1}{2}\log(2\pi)-\frac{1}{2}\log\sigma^2-\frac{1}{2\sigma^2}(x-\mu)^2$$
$$\frac{\partial\log f(x;\mu,\sigma^2)}{\partial\mu}=\frac{1}{\sigma^2}(x-\mu),$$
$$\frac{\partial\log f(x;\mu,\sigma^2)}{\partial\sigma^2}=-\frac{1}{2\sigma^2}+\frac{(x-\mu)^2}{2\sigma^4}$$
$X\sim N(\mu,\sigma^2)$ とすると $Z=\dfrac{X-\mu}{\sigma}\sim N(0,1)$
$$\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\mu}=\frac{Z}{\sigma},\quad \frac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}=-\frac{1}{2\sigma^2}+\frac{Z^2}{2\sigma^2}$$
$$\mathrm{Var}\Big[\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\mu}\Big]=\frac{1}{\sigma^2},$$
$\mathrm{E}\Big[\dfrac{\partial\log f(X;\mu,\sigma^2)}{\partial\mu}\Big]=0,\quad \mathrm{E}\Big[\dfrac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}\Big]=0$ だから
$$\mathrm{Cov}\Big[\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\mu},\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}\Big]$$
$$=\mathrm{E}[\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\mu},\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}\Big]-\frac{1}{2\sigma^3}\mathrm{E}[-Z+Z^3]=0$$
$$\mathrm{Var}\Big[\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}\Big]=\mathrm{E}\Big[\Big\{\frac{\partial\log f(X;\mu,\sigma^2)}{\partial\sigma^2}\Big\}^2\Big]$$
$$=\frac{1}{4\sigma^4}\mathrm{E}[1-2Z^2+Z^4]=\frac{1}{2\sigma^4}\quad\Big(\mathrm{E}[Z^4]=\frac{1}{i^4}\frac{d^4}{dt^4}e^{-t^2/2}\Big|_{t=0}=3\Big)$$